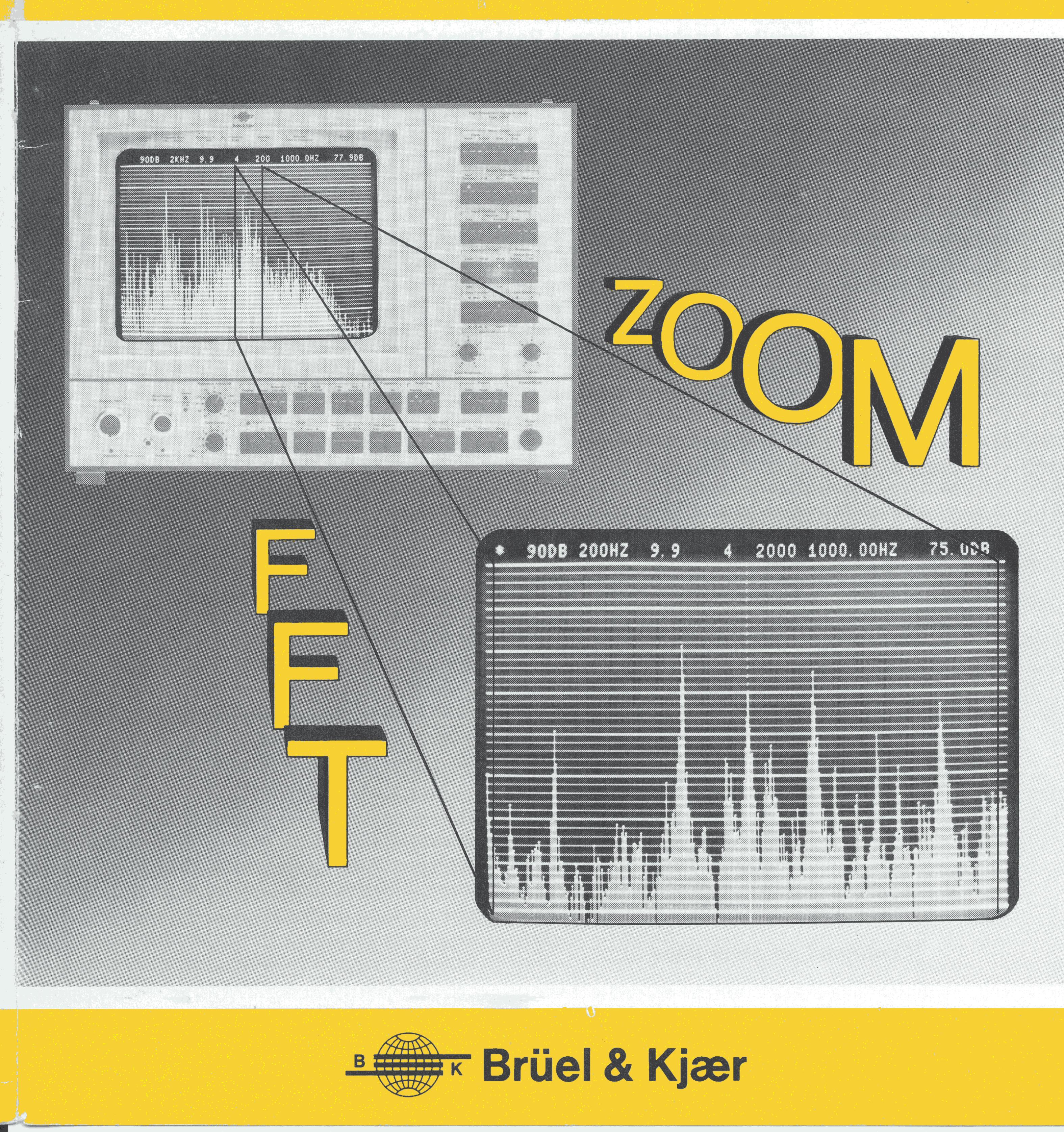
ISSN 0007-2621

No. 2 — 1980°

published quarterly

Leennieal Review

To Advance Techniques in Acoustical, Electrical and Mechanical Measurement



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Contents

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ZOOM-FFT by N. Thrane 3



by

N. Thrane, Ph.D.

ABSTRACT

When ordinary FFT-techniques are used in frequency analysis a spectrum is generated ranging from DC to a given maximum frequency. ZOOM-FFT provides a method by which increased resolution can be obtained within a smaller part of this frequency range. The article discusses the two different principles used today in ZOOM-FFT analyzers. The most important difference between these methods is found in their ability to store and preserve the original time function used for the analysis during the ZOOM-FFT calculations. If the time function is stored new methods become available for the analysis of non-stationary and transient signals. The methods are compared with respect to typical practical applications of ZOOM-FFT analysis of which a number of examples are given.

SOMMAIRE

Pour l'analyse de fréquence avec les techniques FFT habituelles, on obtient un spectre allant du continu à une fréquence maximum donnée. Avec la technique de ZOOM-FFT, on peut analyser avec une plus grande résolution une petite partie de cette gamme de fréquence. Cet article étudie les deux différents principes utilisés de nos jours dans les analyseurs FFT dotés d'un dispositif de ZOOM. La différence la plus importante entre les deux méthodes se situe dans la possibilité d'emmagasiner et de préserver la fonction temporelle originale utilisée pour l'analyse pendant tout le temps où sont effectués les calculs de FFT sur le spectre agrandi à l'aide du ZOOM. Lorsque la fonction temporelle est conservée, on dispose de nouvelles méthodes pour l'analyse des signaux non-stationnaires ou transitoires. Les deux méthodes sont comparées en fonction d'applications pratiques typiques de l'analyse FFT utilisant un ZOOM dont on donne un certain nombre d'exemples.

ZUSAMMENFASSUNG

Bei der Anwendung von gewöhnlicher FFT-Technik in der Frequenzanalyse wird ein Spektrum im Bereich von DC bis zu einer gegebenen maximalen Frequenz erzeugt. FFT mit Fre-

quenzlupe bietet die Möglichkeit, einen Teil dieses Frequenzbereichs mit erhöhter Frequenzauflösung zu analysieren. In diesem Artikel werden zwei verschiedene Prinzipien für FFT-Analysatoren mit Frequenzlupe diskutiert. Der wichtigste Unterschied zwischen beiden Methoden ist das Vermögen, die ursprüngliche Zeitfunktion zu speichern und während der Lupen-FFT-Berechnung zu erhalten. Beim Erhalten der Zeitfunktion eröffnen sich neue Möglichkeiten für die Analyse von nichstationären und transienten Signalen. Die Methoden werden anhand von typischen praktischen Anwendungen, von denen eine Reihe genannt wird, verglichen.

Introduction

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When a time signal is analyzed by use of conventional FFT-techniques a spectrum is produced covering a range from 0 to some chosen maximum frequency, f_{max} . The resolution of the spectrum is determined by the size of the transform, i.e. by the number of samples used to describe the time signal. For a 1 K transform (1024 time samples) the resulting spectrum normally consists of 400 frequency lines evenly spaced over the analysis range. Hence, the line spacing is given by Δf = f_{max} /400, or in terms of the sampling frequency, f_s , as Δf = f_s /1024.

Therefore, when analyzing in a given frequency range, f_{max} , using conventional FFT techniques, the resolution can only be increased, i.e. Δf be decreased, by increasing the transform size.

ZOOM-FFT is a method by which the resolution can be increased without increasing the transform size. However, only a correspondingly smaller part within the original frequency range can be analyzed at a time. The effect of ZOOM-FFT is shown in Fig.1 for a zoom factor of 10.*

Fig.1a shows the result of an ordinary FFT analysis, the so-called *base-band spectrum*. The 400 line spectrum covers a frequency range from 0 to 5 kHz, i.e. the line separation is $\Delta f = 12,5$ Hz. The length of the time signal used for this analysis was T = 80 ms = 1/ Δf . In Fig.1b is

ponential or scan average. Parameters of the line selector (bright line on display) are shown as the last three numbers: selected line number, selected time or frequency, and selected level.

^{*} The photographs shown in this and the following figures were obtained directly from the display screen of the Brüel & Kjær Type 2033 High Resolution Signal Analyzer. Spectra are displayed as 400 lines on a linear frequency axis. The amplitude axis is logarithmic covering a range of 80 dB. In the case of time function displays both axes are linear. The first four numbers of the text line indicate: full scale level (dB RMS re 1 μ V), frequency span of display, samples recorded after trigger, and number of spectra used for linear, ex-

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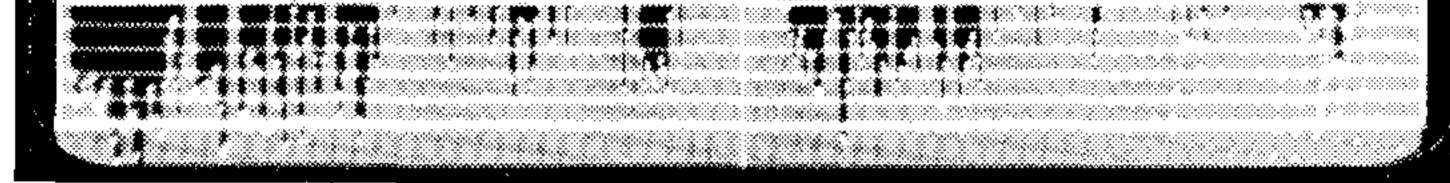


Fig.1. The effect of ZOOM-FFT a) Baseband spectrum 0-5 kHz, b) High resolution spectrum 2900-3400 Hz

shown the result obtained with ZOOM-FFT, the *high resolution spectrum*. The 400 line spectrum now covers a frequency span of 500 Hz, i.e. 1/10 of the range of the baseband spectrum, from 2900 Hz to 3400 Hz, and accordingly the line separation is decreased to $\Delta f = 1,25$ Hz. However, the length of the time signal required for this analysis was T = 800 ms = $1/\Delta f$.

The term "zoom" has, of course, been taken over from photography and indeed the meaning is just the same: a small part of the "object" is blown up to fill the full size of the frame, thereby allowing more details

to be seen. But we can carry this analogy even further when we look at how large zoom factors can be used in practice.

In photography the resolution is in principle determined by the lens aperture and focal length. However, for a given aperture, "zooming in" will decrease the illumination of the film and longer exposure times will be required. In order to obtain the maximum resolution allowed by the lens system it is therefore essential that the object itself is stationary during the increased exposure time. If the object moves or changes in some other way we will not get the expected resolution, but instead an unsharp "smeared" picture caused by the non-stationarity of the object itself.

The same applies to frequency analysis. The higher resolution, or the more narrow bandwidth we require in the analysis, the longer will be the required time signal. In the example of Fig.1, the resolution was changed by a factor of 10, from $\Delta f = 12,5$ Hz to $\Delta f = 1,25$ Hz. Accordingly the length of the time signal needed for the analysis was increased from T = 80 ms to T = 800 ms. In this case the signal was sufficiently stationary over these 800 ms and the high resolution spectrum indeed gave a much more detailed spectrum compared to the baseband spectrum.

The inverse relationship between Δf and T is a result of the so-called "Uncertainty Principle", which states that the product of Δf and T is always larger than, or equal to, one, $\Delta f \times T \ge 1$. Although this principle is one of the fundamental properties of the Fourier Transform itself, the practical consequences often come as a surprise. The use of ZOOM-FFT with very large zoom factors might result in extremely long analysis times and therefore require the utmost with respect to signal stability.

ZOOM-FFT can be implemented in two different ways, both of which are completely digital in nature and therefore have all the advantages of digital techniques with respect to stability, dynamic range, linearity, etc.

Method 1: ZOOM-FFT by frequency shift (heterodyning) and low-pass filtration.

Method 2: ZOOM-FFT by recording of a long time signal and trans-

forming it by parts using a smaller transform.

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Until recently only method 1 was commercially available. But with the

introduction of the Brüel & Kjær Type 2033 'High Resolution Signal Analyzer' method 2 is also available.

In the following we shall discuss both methods, their advantages and disadvantages. However, since method 2 as used in the Type 2033 is both new and unique we shall put special emphasis on some of the new analysis methods which are made possible, especially with respect to the analysis of transient and non-stationary signals.

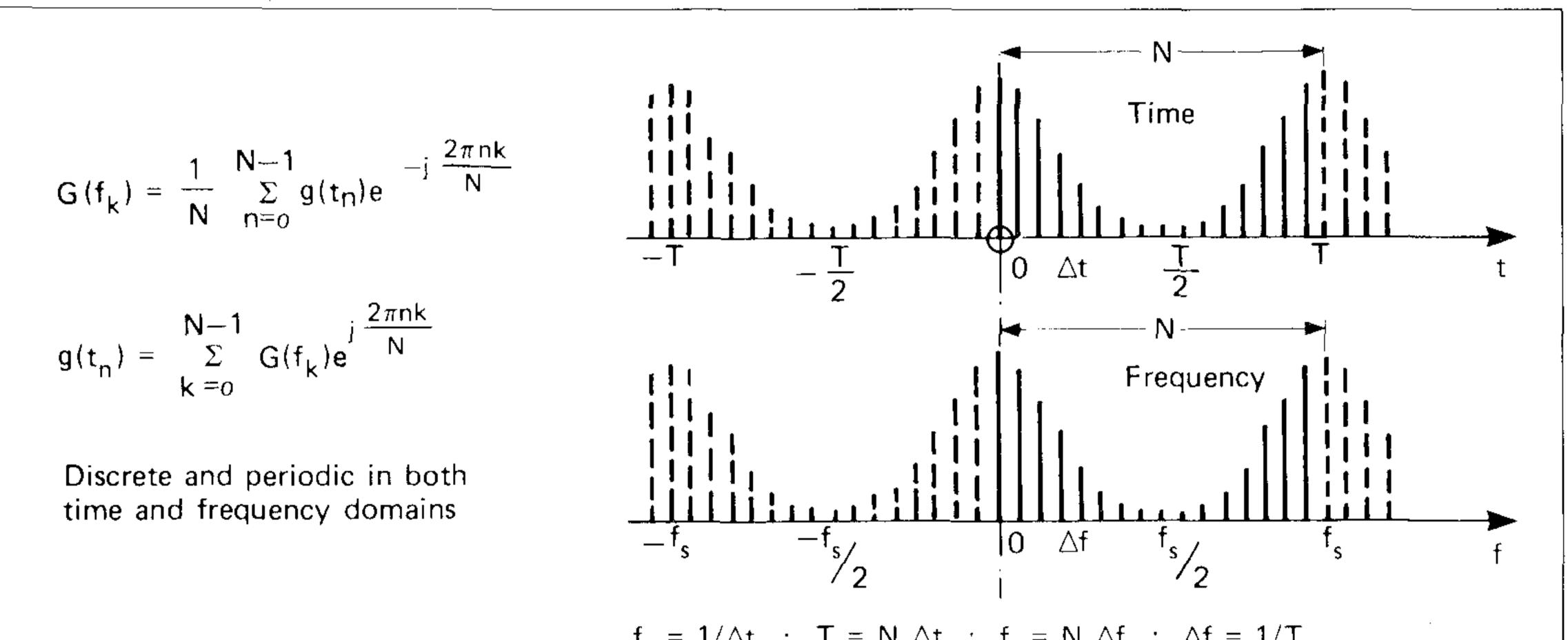
The Fast Fourier Transform (FFT)

The FFT algorithm describes a very fast and efficient computation of the so-called Discrete Fourier Transform (DFT). This special version of the general Fourier transform has been discussed in detail elsewhere (Ref.1) and only a brief summary will be given bere. A general discussion of frequency analysis can be found in Ref.2.

It is important to realize that the DFT performs a blockwise analysis of the time signal. One block consisting of N samples of time data is first recorded. For a sampling frequency of f_s , the sampling interval is $\Delta t = 1/f_s$ and the block corresponds to a record length of $T = N\Delta t = N/f_s$. However, in the analysis the DFT assumes that this block is one period of an artificial infinitely long periodic signal. Accordingly, the frequency spectrum will consist of evenly spaced lines along the frequency axis, corresponding to the various harmonic frequencies of the time signal.

This is shown in Fig.2.

As also can be seen from Fig.2, both the time signal and the frequency



$$\mathbf{r}_{s} = \mathbf{I} / \Delta \mathbf{t}$$
; $\mathbf{I} = \mathbf{N} \Delta \mathbf{t}$; $\mathbf{r}_{s} = \mathbf{N} \Delta \mathbf{f}$; $\Delta \mathbf{f} = \mathbf{I} / \mathbf{I}$
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Fig.2. The Discrete Fourier Transform

spectrum are periodic as well as discrete, the period in the frequency spectrum being f_s , and containing N evenly spaced samples. The spacing of the frequency lines, i.e. the frequency resolution, is therefore given by $\Delta f = f_s/N = 1/T$, in agreement with the uncertainty principle mentioned above.

If the time function is a real-valued function, which is normally the case, the frequency spectrum becomes a conjugate even function, meaning that the components at + f and —f have the same amplitude but opposite phase. Hence, only N/2 of the frequency components are independent, namely, those from 0 to $f_s/2$. Furthermore, in order to prevent aliasing of frequency components an analog antialiasing filter with cut-off frequency below $f_s/2$ has to be used, leaving less than N/2 of the frequency components unaffected. In commercial FFT analyzers, N is often chosen as 1 K (= 1024), giving 512 independent frequency lines, of which the lowest 400 are valid. This gives the relationship between the measurement frequency range f_{max} (line 400) and the sampling frequency f_s (line 1024) as $f_s = 1024/400 \times f_{max} = 2.56$

The artificial periodic signal being analyzed by the DFT can be considered as a repetitive playback of that particular part of length T which

Frequency 0,1 0,2 0,4 0,6 0,8 1 x 1 2,0 4,0 6,0 8,0

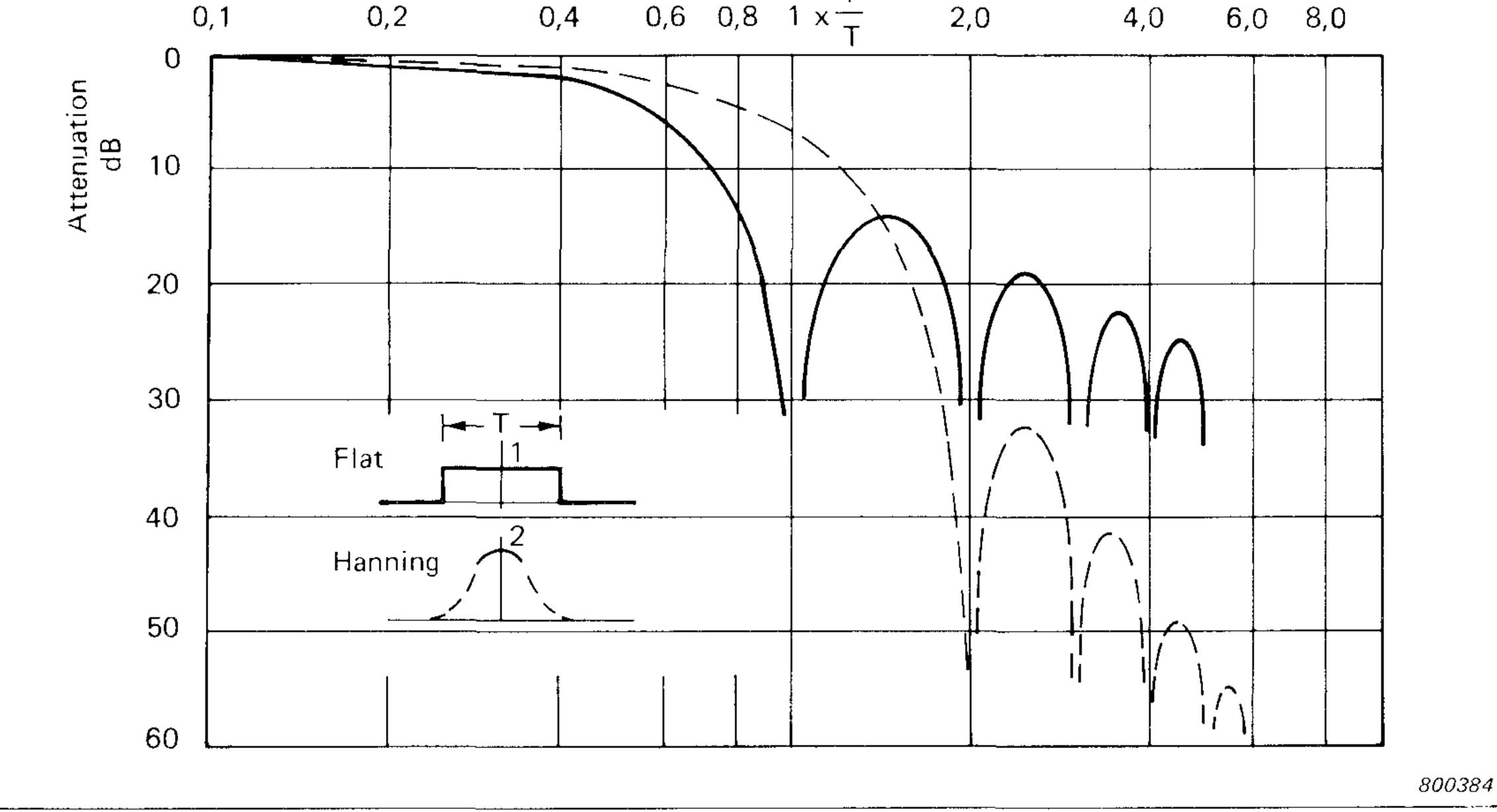


Fig.3. Comparison of spectra (filter characteristics) of the rectangular and the Hanning Weighting functions

happened to be recorded. Therefore, if the original signal was long -"continuous" - compared to T we might encounter discontinuities or "clicks" whenever the periodic signal repeats itself. In order to avoid the influence of such discontinuities in the spectrum and obtain a better selectivity, a smooth time weighting function has to be used, e.g. the "Hanning" function shown in Fig.3. The effect of multiplying the recorded time signal by this weighting function is to decrease the amplitude of the resulting signal to zero at the beginning and the end of the record length, thereby eliminating any discontinuities.

However, for transient signals shorter than the record length T, the Hanning weighting should not be used since it could heavily distort the signal and result in a false analysis. Besides, in this situation there would be no need for the Hanning weighting since there would be no risk of discontinuities. In this case the "Flat" weighting function, also shown in Fig.3, should be used.

The Hanning function is simply one period of a sinusoid having a length equal to the record length, T, but lifted up so that it starts and stops at zero, having an amplitude at the centre equal to 2. Fig.3 also shows the filter characteristics corresponding to the two weighting functions. The amplitudes are equal at the centre, meaning that sinusoidal signals will be analyzed with the same amplitude independent of the weighting function. Note the much better selectivity of the Hanning function compared to the Flat function. Table 1 summarises the main features of the two weighting functions. The difference in selectivity is indicated by the big difference in the slope of the sidelobes. Of great importance is also the difference in noise bandwidth, that of the Hanning function being 50% larger than that of the Flat function, which is equal to the line spacing Δf .

Name	Noise bandwidth	Highest sidelobe	Sidelobe fall-off rate							
Flat	1 x ∆f	—13 dB	20 dB/decade							
Hanning	1,5 x ∆f	32 dB	60 dB/decade							
	$\Delta f = 1/T = line separation in sampled spectra$									

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Table 1. Comparison of window functions

Since the DFT is analyzing a periodic signal, the amplitudes of the various frequency components will indicate the power of these components, the unit being amplitude squared, e.g. Volt². Depending on transducer and conditioning preamplifier, the Volt unit can be directly replaced by an engineering unit such as mm, m/sec, g, Pa, etc. This causes no problems when analyzing deterministic signals, i.e. those made up of sinusoids, where the correct analysis unit is power.

However, when analyzing random or transient signals corrections have to be made. For random signals the correct analysis unit is power density, i.e. power per Hz. Hence the DFT measured power levels have to be divided by the bandwidth, which depends both on the frequency range and also on the weighting function. For a given signal having a given power density an analysis using Hanning weighting will give a power per spectral line which is a factor of 1,5 or 1,76 dB higher than the analysis using the Flat weighting.

For transient analysis the correct analysis unit is energy density. In this case, the DFT measured power levels have to be divided by the analysis bandwidth and multiplied by the record length in order to give the right unit. Both these parameters depend on frequency range, but normally only the Flat weighting function will be used. However, as discussed later, the scan averaging feature of the Type 2033 allows the Hanning weighting to be used for transient analysis, and in this case both bandwidth and effective record length will also depend on the weighting function

function.

Zoom - FFT

As discussed above, ordinary FFT always produces a spectrum ranging from zero frequency to some maximum frequency, f_{max} . For a given transform size, for example N = 1024, the resolution was given as Δf = 2,56× $f_{max}/1024$. Hence, better resolution can only be obtained in one of two ways:

- 1) Either by decreasing f_{max} , and losing the high frequency information, or
- 2) by increasing the transform size, which will require larger memories and longer calculations.

When using ZOOM-FFT the increasd resolution can be obtained without losing the high frequency information or increasing the transform

size. However, only a small part of the original frequency range can be analyzed at a time.

The two ZOOM-FFT methods mentioned previously are closely related to the two possibilities shown above, since in the final analysis they are both based on ordinary FFT. But they have one thing in common: namely, the limitation set up by the uncertainty principle, $\Delta f \times T \ge 1$.

If we want to perform a frequency analysis with a, say ten times smaller value of Δf , we *have* to base this analysis on a 10 times longer time signal. The two methods, to be discussed now, differ with regard to the way in which this longer time signal is treated.

Zoom-FFT by frequency shift — low pass filtration

This type of ZOOM-FFT is the one normally implemented in FFT analyzers. The increased resolution is obtained by shifting (heterodyning) the frequency span of interest to fall around zero frequency, followed by a reduction of frequency range. In Fig.4 is shown a block diagram of the data flow, while the various stages of the process can also be followed in terms of frequency in Fig.5 and Fig.6.

Following Fig.4, the signal is low pass filtered by the antialiasing filter before being digitized in the ADC. This ensures that the sampling theorem is obeyed, i.e. that the highest frequency component in the signal

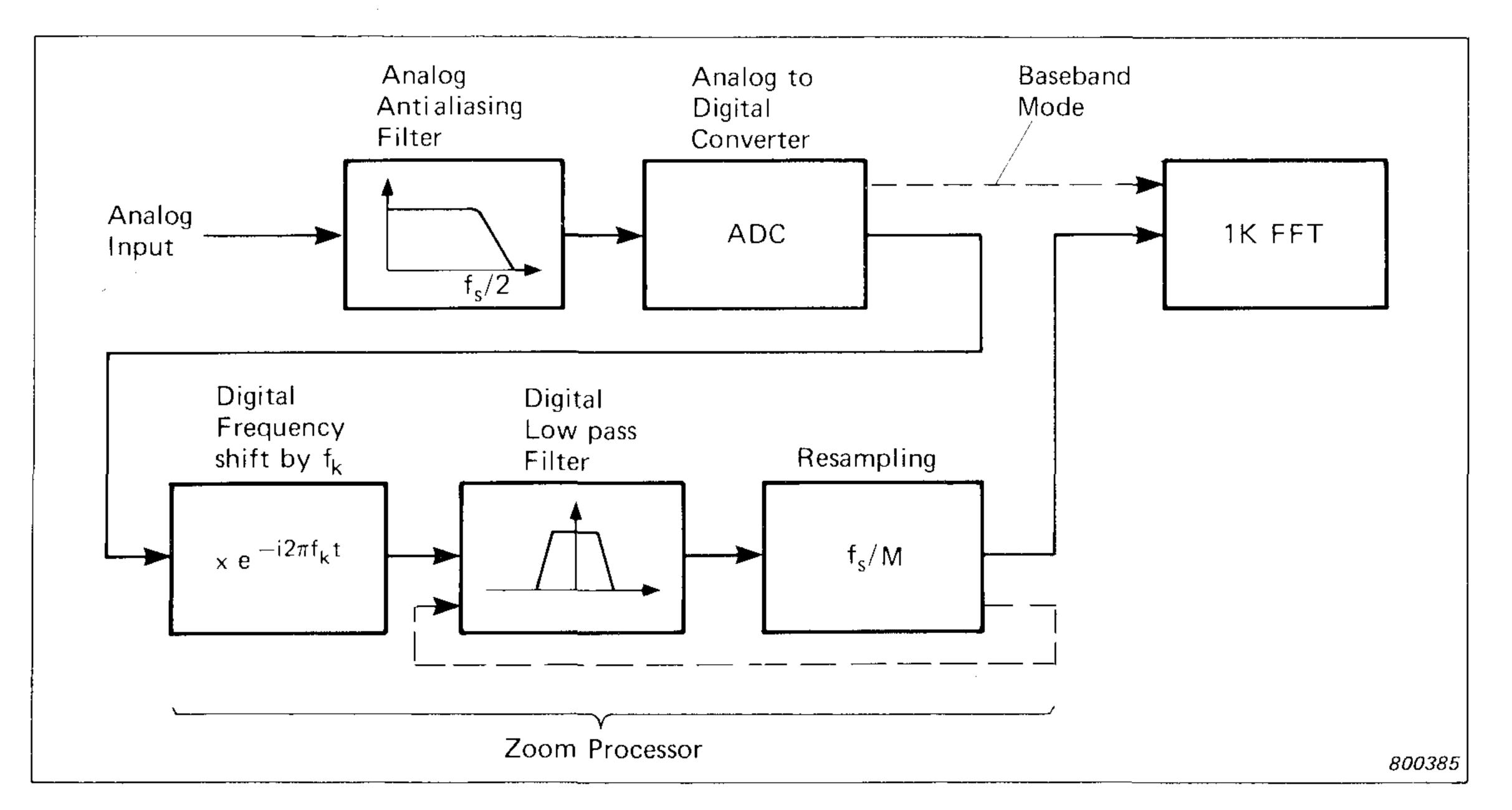
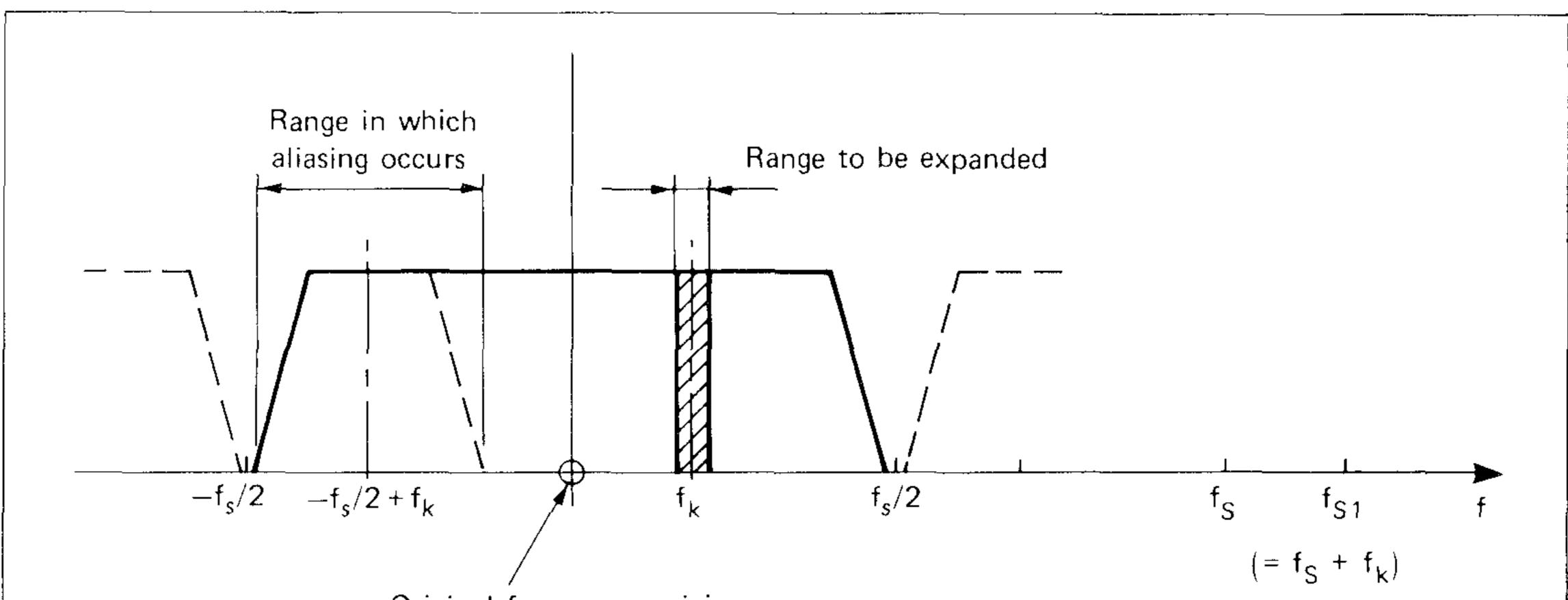


Fig.4. Data flow for ZOOM-FFT using digital frequency shift—low pass filtration



Original frequency origin

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Fig.5. Frequency shift caused by multiplying signal by unit vector rotating at $-f_k$

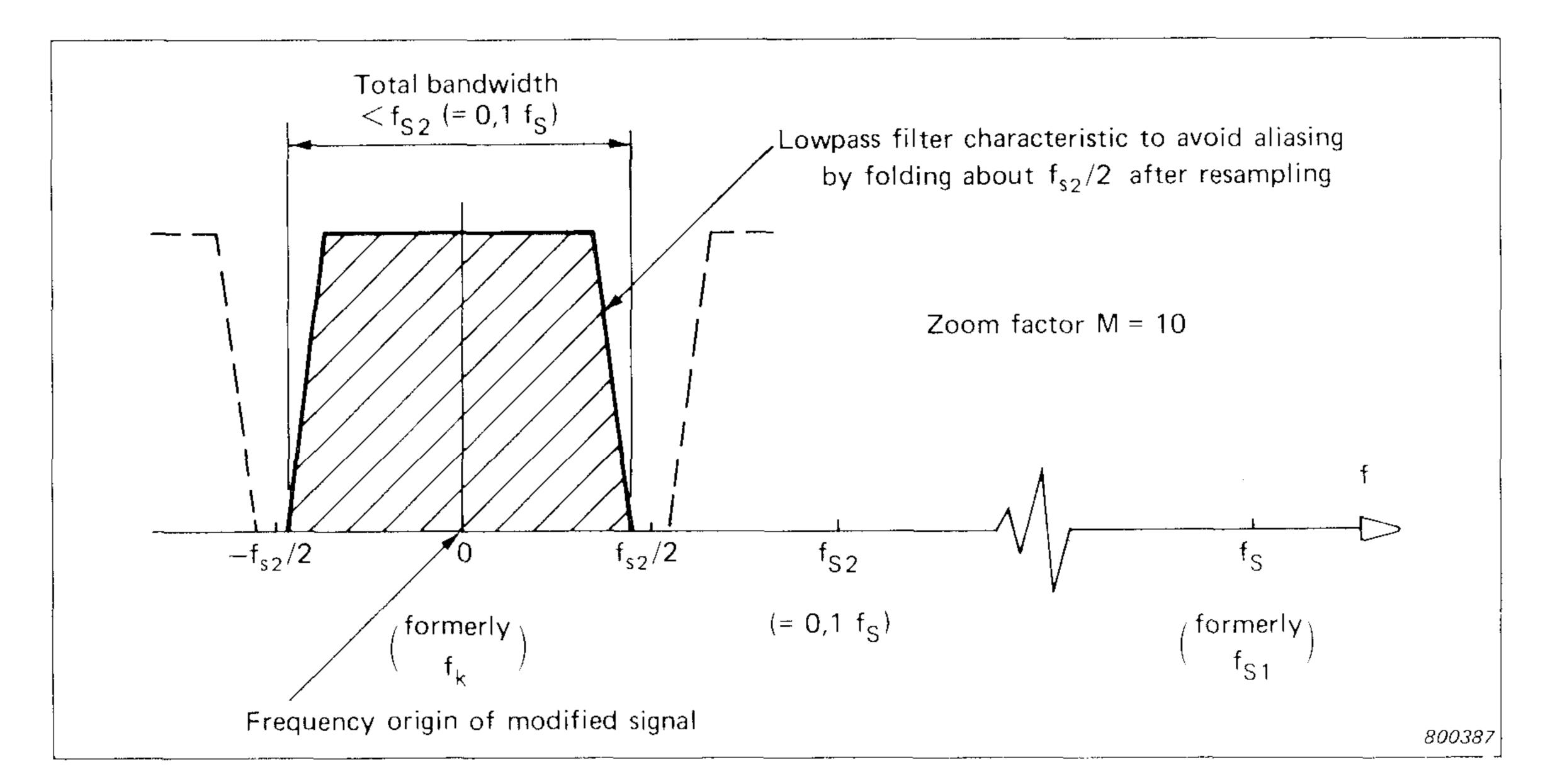


Fig.6. Detail of range to be expanded after resampling

is sampled with more than two samples per period. The samples from the ADC (sampling frequency, f_s) are then fed to the zoom processor where they are first multiplied by a complex exponential function, $e^{-i2\pi f_k t}$, i.e. by a unit vector rotating at $-f_k$. This multiplication performs a frequency shift of the signal of $-f_k$. Hence, the frequency span of interest is shifted from the vicinity of f_k to around DC, see Fig.5 and Fig.6.

After this frequency shift, the time samples are fed through a digital low pass filter and a resampling process, the function of which we shall discuss only briefly.

Let the zoom factor be M. The purpose of the digital low pass filter is to pass only the frequency span of interest around DC (originally f_k), see Fig.6. This process also ensures that the filtered signal now will have more than 2M samples per period at its highest frequency component, i.e. the signal will be oversampled by a factor of M.

It is therefore possible to reduce the sampling frequency by a factor of M without losing any information. During the resampling process only 1 out of M samples is led through, the rest being discarded.

Finally, the samples are fed to an ordinary 1 K sample FFT analysis. Since the FFT, like all other digital analyzers, performs its analysis relative to the sampling frequency, the analysis range, f_{max} , has indeed been reduced by a factor of M, but now centered around f_k . A ZOOM-FFT spectrum is produced.

Notice that the 1 K samples being analyzed really represent information from M times 1 K samples of the original time signal, i.e. zooming with a factor of M involves an M times longer signal, as required by the uncertainty principle.

The digital low pass filter of Fig.4 also works relative to the sampling frequency at its input. It is therefore possible to recirculate the data after resampling in order to obtain further zoom function. This is shown by the dashed line in Fig.4. For M = 2 each pass will increase the total zoom factor by 2. In this way large zoom factors can easily be obtained in a binary sequence by relatively simple means.

If the zoom processor is to function it has to work in real-time, i.e. it has to handle the samples from the ADC as fast as they come. This requires fast multiplications to be available both in the frequency shift part and in the digital filter. The overall real-time performance will then be determined by the FFT calculation itself. However, due to the reduction of the sampling frequency, it will now be the frequency *span* of the analysis which determines the real-time performance.

The main features of this type of ZOOM-FFT are:

Large zoom factors Real-time zoom analysis

Original time function not stored.

Since the pro's and con's on this list are exactly opposite to those of

the other method to be discussed, we shall postpone a discussion till later.

Zoom-FFT by recording the time signal

This type of ZOOM-FFT is the one implemented in the Brüel & Kjær Type 2033 High Resolution Signal Analyzer. We shall discuss this method specifically for a zoom factor of 10, referring to Fig.7 which shows the data flow of the 2033.

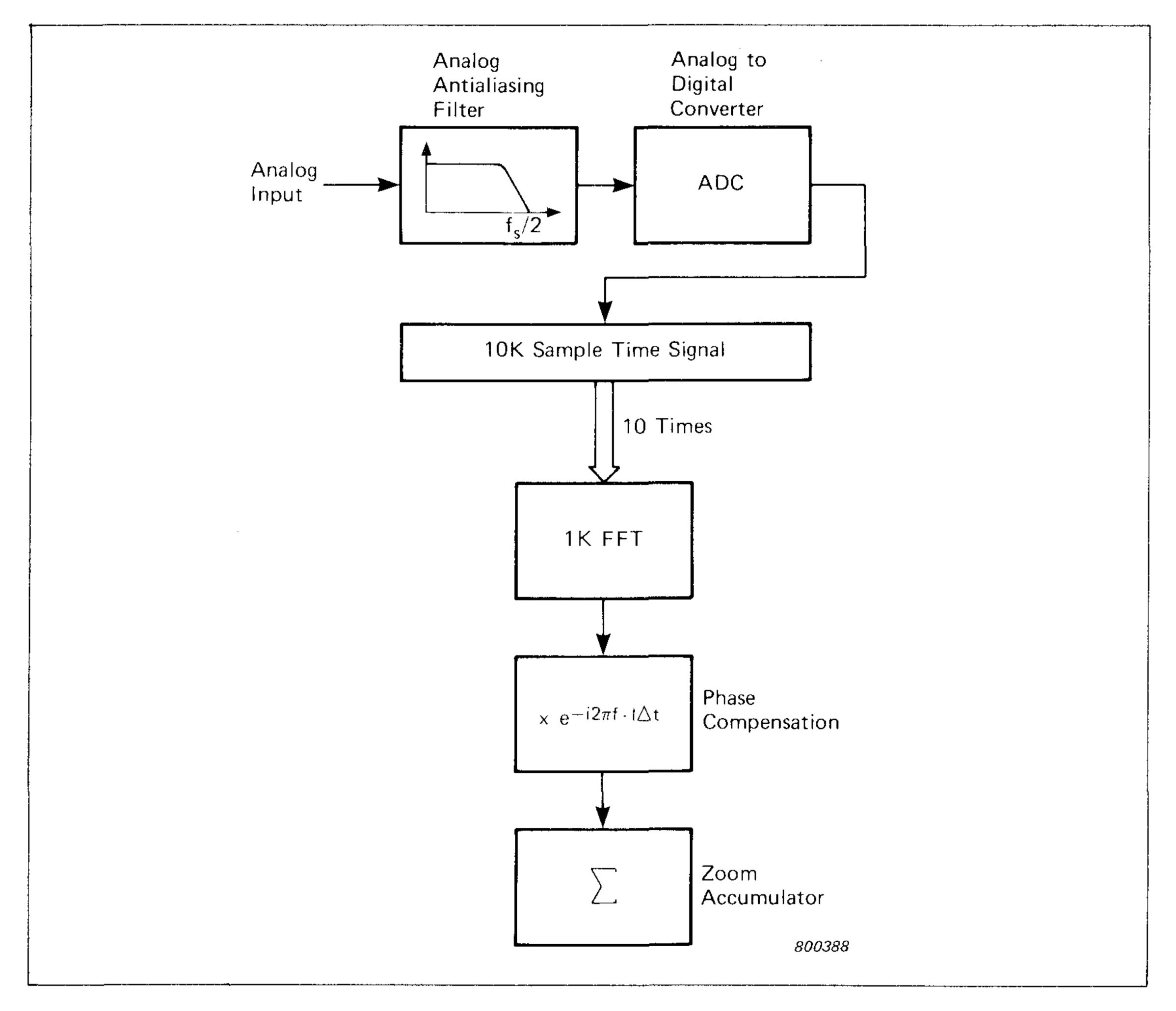


Fig.7. Data flow of the B & K Type 2033 'High Resolution Signal Analyzer''

The increased resolution is obtained by first recording the necessary duration of time signal, i.e. 10 K samples. Then an ordinary 1 K FFT analysis is performed 10 times on different parts of the 10 K samples, combining the individual results into a 400 line high resolution spectrum in the zoom accumulator.

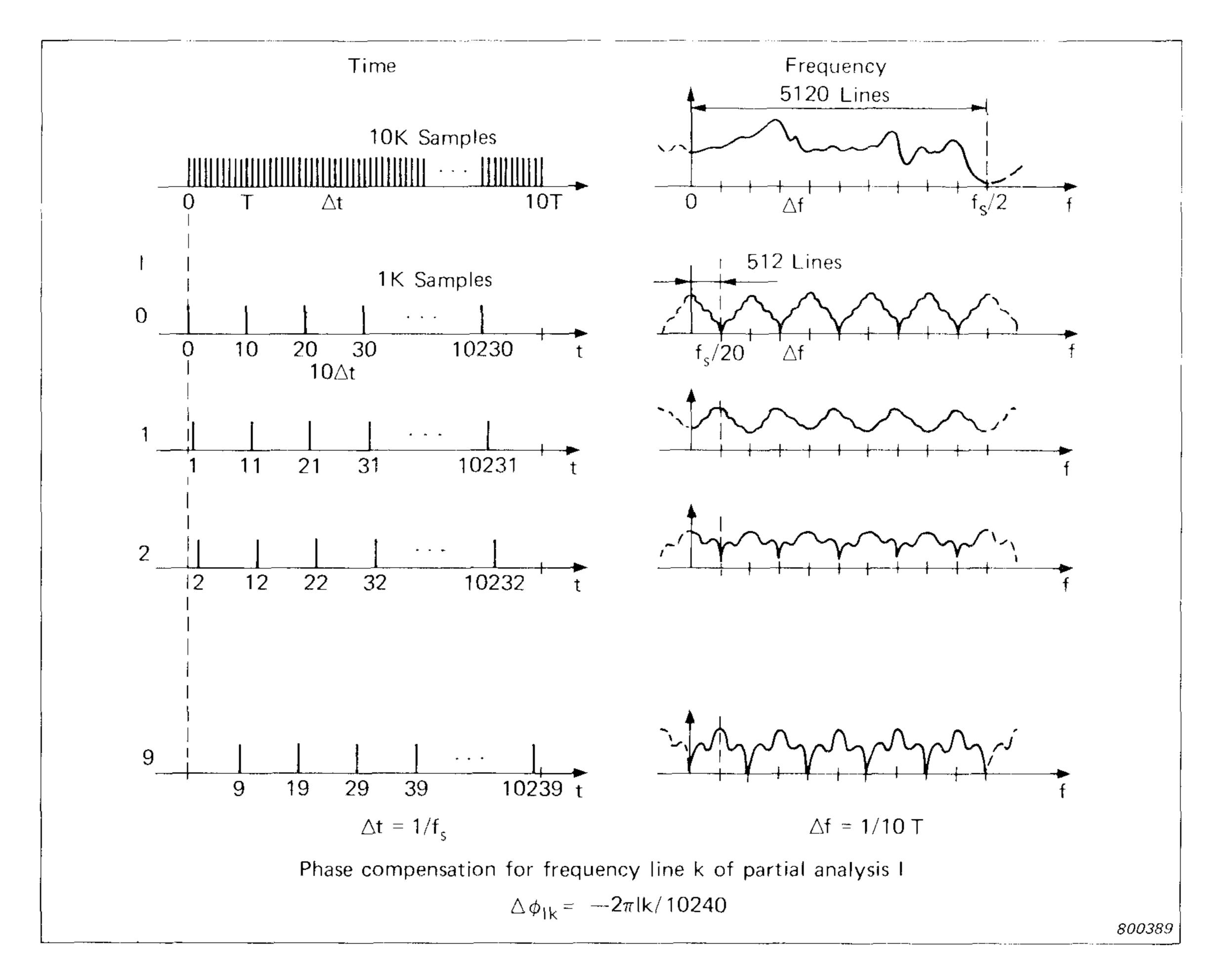


Fig.8. Implementation of a 10 K FFT using a 1 K transform ten times

Each of the ten 1 K time records being transformed consists of a ten times undersampled version of the full 10 K record, see Fig.8. This means that the first 1 K record consists of samples number 0, 10, 20, ..., 10230, the second record of samples number 1, 11, 21, ..., 10231, and the last record of samples number 9, 19, 29, ..., 10239. Hence, each of the partial records covers the full record length. It is seen that the sum of the ten partial time records is equal to the original 10 K time record. Since the Fourier transform is a linear transform the sum of the full 10 K time records must also be equal to the spectrum of the full 10 K time record.

When transforming each of the ten partial records the FFT assumes the time zero to be associated with a particular sample, normally the first. However, only the first partial record (I = 0) has its first sample coinciding with the first sample of the 10 K record. The others have their first samples at a time equal to $I \times \Delta t$ where Δt is the original sampling interval. Therefore, before adding the partial frequency spectra it is necessary to compensate for this time shift between the individual records.

In the frequency domain the time shift corresponds to a phase shift of $\Delta \phi = -2 \pi f (1 \times \Delta t)$ or in terms of frequency line number, k, $\Delta \phi = -2 \pi I k / 10240$. Hence, each of the frequency lines in each of the partial spectra has to be phase compensated before entering the zoom accumulator, as indicated in Fig.7.

In Fig.9 is shown a computational block diagram of the 2033 zoom calculation. The ten 1 K FFT of the partial time records, the phase compensation factors $W_{10240}^{l \ k}$ and the final summation can be easily recognized.

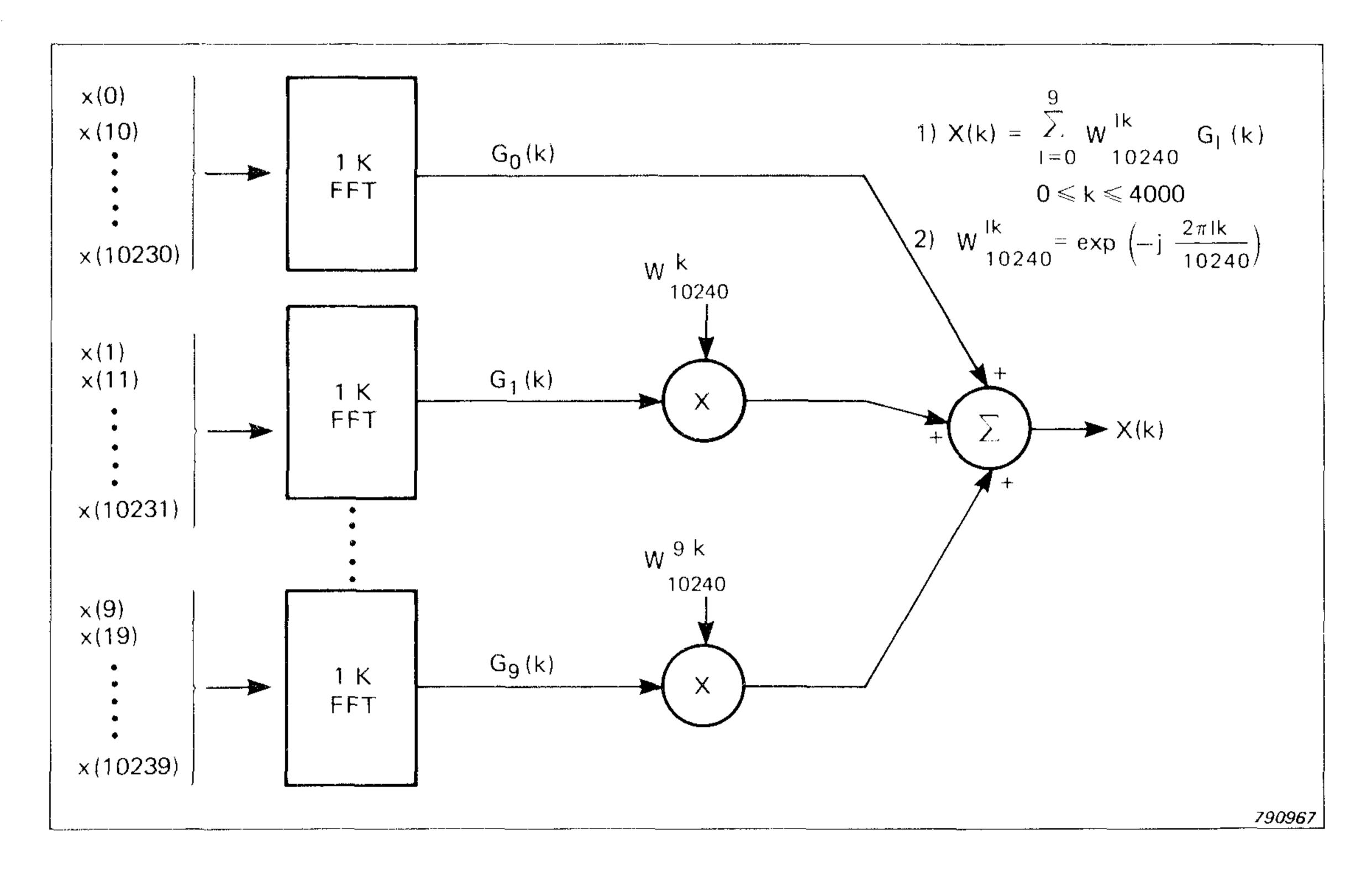


Fig.9. Block diagram representing the 2033 zoom calculation

The frequency spectrum of the 10K sample time record consists of 5120 independent frequency lines. Due to the cut-off of the analog antialiasing filter, however, only the first 4000 lines are valid. Although the 2033 only calculates 400 lines in each zoom analysis, the original 10K time record is still available and by repeating the zoom transform 10 times in contiguous frequency spans, a full 4000 line spectrum can be obtained without rerecording the time signal. This feature is in contrast to the previous method, where the frequency span had to be se-

lected before the time function was recorded, and hence the time function had to be rerecorded each time a new frequency span was required.

It is interesting to notice in Fig.8 that each of the partial time records is heavily undersampled, and therefore aliasing of frequencies appears in the partial spectra. Each partial spectrum consists of only 512 independent frequency lines, but as mentioned previously each spectrum is a conjugate even function and periodic over $f_s/10$ or 1024 lines. However, when the ten partial spectra are added, the aliased components nicely cancel, leaving only the true components in the final result.

The main features of this type of ZOOM-FFT, as implemented in the 2033, are:

Zoom factors ≤ 10 Non real-time analysis Storage of original time function

The factor which in practice determines the first two features is limited memory space. Large zoom factors would require the recording of correspondingly longer time records. Real-time analysis would require two parallel input memories, so analysis could take place in one memory while recording took place in the other. It is felt that these drawbacks are more than compensated for by the preservation of the original time function.

Comparison of ZOOM - FFTs

A comparison of the two different types of ZOOM-FFT should be based on the practical use of FFT analyzers. One major application is in the study of vibration from rotating machines, where the narrow constant bandwidth filters and the linear frequency scale of the FFT enable a fast identification of important components and an easy recognition of harmonic patterns and sideband structures. Another important application is in the analysis of transients, since the input stage of an FFT analyzer in effect is a transient recorder.

We shall now discuss the three main features of the zoom transforms.

Large Zoom Factors are required whenever a very narrow band analysis is needed. However, very long time records also have to be used in the analysis and therefore the stability of the signal source itself has to be extremely good. With a moderate zoom factor of 10 smearing of the spectral peaks will be observable in the upper end of the frequency range for frequency variations of the order of 0,05%. For higher zoom

factors the variations should be correspondingly smaller. It is likely that very high zoom factors are only beneficial when analyzing electrically generated signals. Due to the smearing effect they are seldom necessary in measurements of vibration of rotating machines. Even in gearbox analysis a zoom factor of 10 will allow the detection of side-band structures around the third harmonic of the tooth meshing frequency for gears having more than 200 teeth.

Real-Time Analysis is essential when dealing with non-stationary, rapidly varying signals. In practice, most of these types of signals are found in the field of acoustics, such as in reverberation measurements and pass-by noise. However, in these cases a real-time 1/3 octave filter analyzer will be preferred anyway. Furthermore, according to the uncertainty principle, rapidly varying signals are generally broad-band and do not require a narrow band analysis. Real-time analysis is therefore seldom a condition of narrow band vibration measurements, although fast analysis is an advantage. Even in transient analysis only the initial recording of the signal must take place in real-time, not the analysis itself.

Storage of the original time function. It might be said that the most important feature of ZOOM-FFT found in practice is its ability to handle very long time records, rather than performing very narrow-band analysis. This is obviously of the utmost importance when dealing with the analysis of transient signals. The advantages of storing the original time function can be grouped as follows:

- a) The ability to perform multiple zoom transforms from the same time signal without the need of rerecording the signal.
- b) Scan analysis of non-stationary signals and scan average of transients. These features will be discussed in the next section.

The limitations of this method with regard to large zoom factors and real-time have been discussed above.

One consequence of the multiple zoom feature is that 10 contiguous 400-line zoomed spectra can be generated from the same 10 K sam-

ples, allowing a 4000-line spectrum of a single time record to be obtained. This spectrum will extend across the entire base-band frequency range.

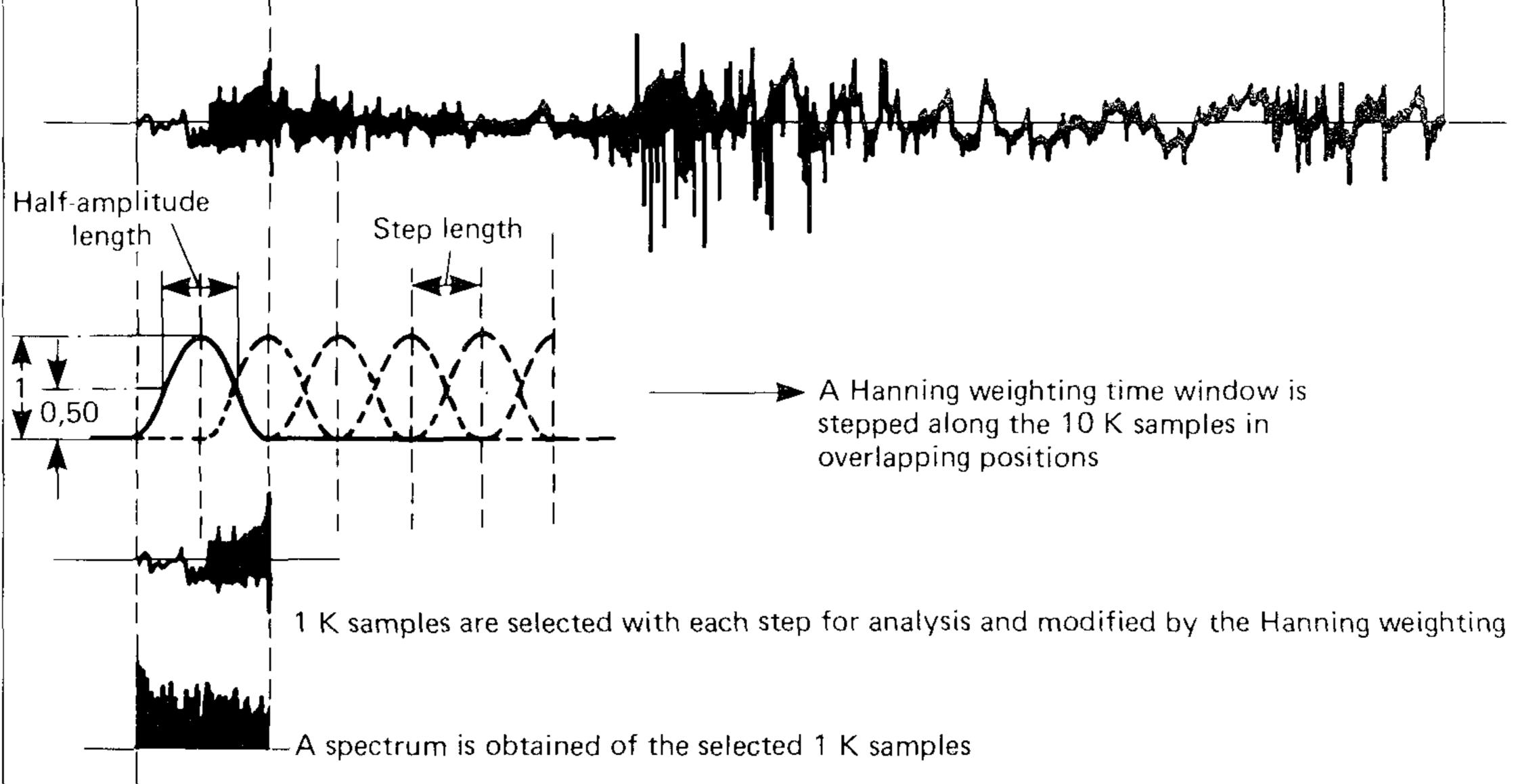
Another consequence is related to processing times. An example will clarify this. Let us compare the two types of ZOOM-FFT in a 10 times zoom analysis on a 0 to 10 Hz base-band frequency range. Both methods will require 400 s of time data. Thereafter the 2033 can make as many zoom transforms as required from the same data, each new analysis taking about 1 s. The real-time zoom transform, however, will require a new 400 s of data for each new analysis. In this extreme case the real-time zoom transform will be much slower than the non-real-time zoom analysis.

Scan Analysis - Scan Average

The scan analysis and scan average features are not related to the zoom analysis, but rather to the fact that the recorded time signal is ten times longer than the record length used by the FFT. Since both features are new in FFT analysis and unique to the 2033, we shall discuss their properties in detail and with specific reference to their implementation in the 2033.

The operation of the scan analysis is shown in Fig.10. A 10 K sample time signal is first recorded and stored in the 2033 input memory. A 1 K sample long time window - Hanning or Flat - can then be stepped

]
	Non-stationary Time Record	
	(e.g. in 10 K memory)	



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Fig.10. Illustration of the scan analysis where a 1 K sample time window is stepped along the 10 K sample time function

along the 10 K samples - either manually or automatically - while producing a new 400-line baseband spectrum for each step. The scan process is non-destructive and can be repeated on the same time signal using different step size or weighting functions.

In a *manual scan* the required 1 K sample time signal is selected by manually positioning it on the display screen, and then selecting display of the spectrum.

In an *automatic scan* the window will be stepped across the entire 10 K samples, producing and displaying a new spectrum for each step. The step size can be selected in a binary sequence from 1024 samples down to 8 samples, and the number of steps per scan will vary from 10 up to 1153. Each step has a duration of approximately 110 ms, so the total scan time can be selected from about 1 sec to about 2 min. These parameters are shown in Table 2.

Scan	Analysis			n Average ng Weighting)
Scan step (samples)	Number of spectra per scan	Scan time	BT product	Effective record length (K samples)
1024	10	1s	10	
512	19	2s	18,5	
256	37	4 s	22	9 1/4
128	73	8s	22	9 1/8
64	145	15s	22	91/16
32	289	30s	22	9 1/32
16	577	1 min	22	9 1/64
8	1153	2 min	22	9 1 /128
				800132

Table 2. Parameters of Scan Analysis and Scan Average using Hanning Weighting

The automatic scan function is of exceptional value in the analysis of non-stationary signals such as speech, music, fast machine run-ups, etc. The effect of the automatic scan is to produce a "slow motion" display such that rapid changes in the spectrum can be seen and followed as the time window moves along the entire time signal. Otherwise as in the case of a real-time analysis, these changes might occur so rapidly that their visual comprehension become impossible.

Signals from 200 ms to 400 s duration can be analyzed in this manner, depending on the baseband frequency range. Using a 5 kHz baseband frequency range typical of speech analysis, the record length is 800 ms. In this case the Hanning weighting function would have an effective length of about 30 ms and could be displaced along the signal in steps as small as 0,7 ms.

When an automatic scan is performed by the 2033, it simultaneously produces a *scan average*, this being a linear average of the spectra generated by the scan. The scan average is used to obtain a 400 line averaged baseband spectrum from the 10 K sample time function, rather than the 4000 line spectrum produced by the zoom function. In this respect it can be used with both continuous and transient signals.

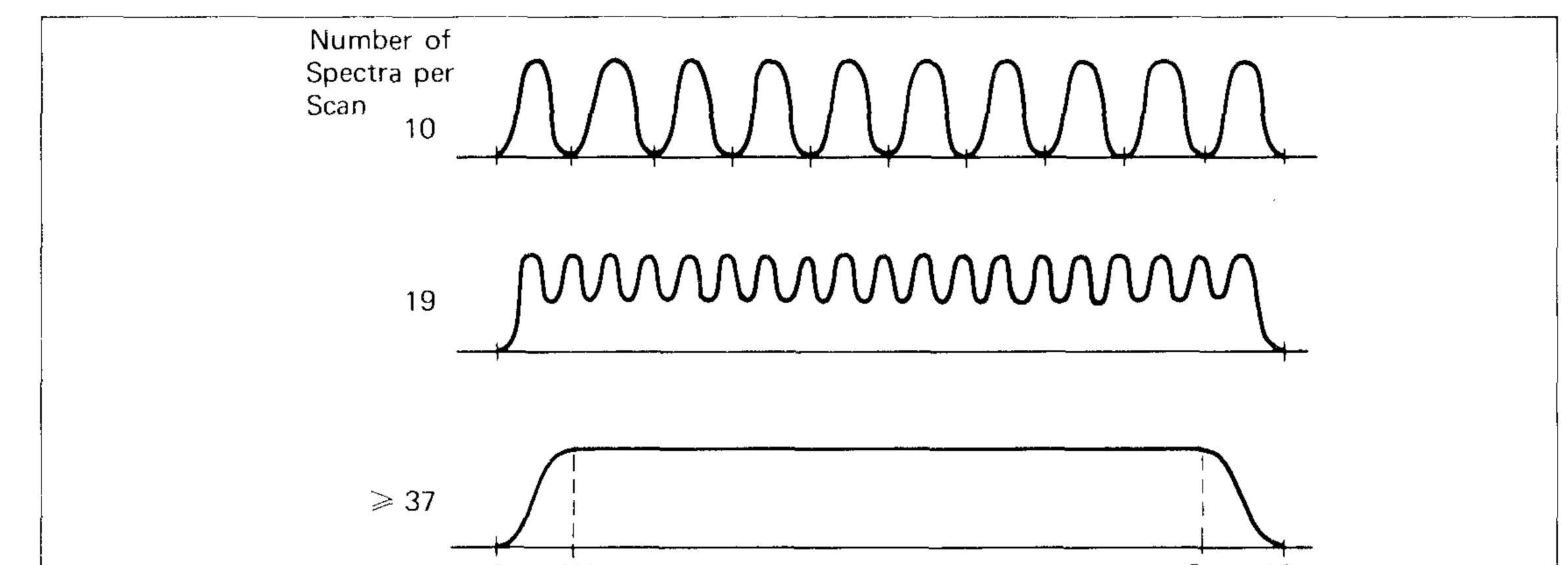
When analyzing continuous signals using scan average, the BT product of the average spectrum will depend on the amount of overlap between the individual spectra. In Table 2, the values of BT products are given when the Hanning weighting function is used. Note that a BT product of 22 can always be obtained, provided the signal fills the entire 10 K sample memory. This will also be the case when the analysis is performed in the higher frequency ranges where it is not working in realtime.

An important application of the scan average is the analysis of transient signals, which can be contained within the 10 K sample memory. The alternative analysis method would be use of the zoom analysis to produce a 4000-line spectrum. Since transients tend to be broadband, this 4000-line spectrum would include a large amount of redundant data. With the scan average only a 400-line spectrum is produced covering the same frequency range.

When using scan average for transient analysis it is essential to ensure that all parts of the signal are analyzed with equal weight, i.e. that the overall time weighting function is flat. Furthermore, breaking the signal up into smaller parts should be done in a way which does not distort the signal. Both of these requirements are met if the Hanning weighting is used and the number of spectra per scan is larger than or equal to 37.

In Fig.11 is shown the overall weighting functions for various overlaps

of the Hanning weighting function. Notice that only the central 8 K samples of the time function have a completely flat weighting for $N \ge 37$. In



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Fig.11. Overall weighting functions of scan average using Hanning Weighting

practice, the central 9 K samples could often be used without significant influence on the result.

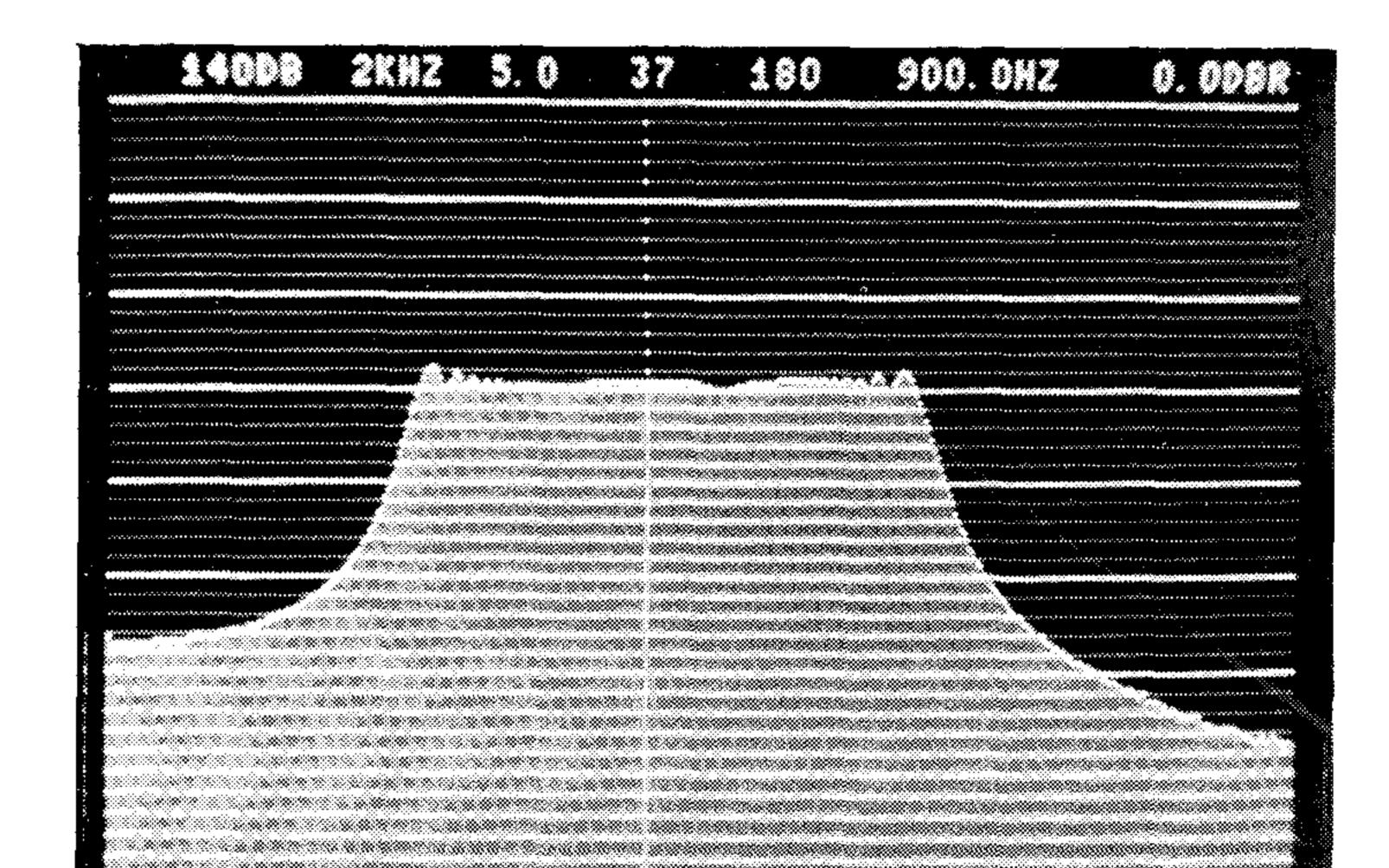
For calibration of scan average spectra of transients the effective record length, T_{eff} , of the analysis has to be known. These values are also shown in Table 2. Notice that T_{eff} is somewhat shorter than the full 10 K sample memory giving rise to a minor correction of about 0,4 dB when scan averaged spectra are compared to zoomed spectra. A more detailed discussion of this subject will be found in the appendix.

An example is shown in Fig.12. A sine sweep from 500 Hz to 1500 Hz with a duration of 0,3 s was captured in the 10 K sample input memory of the 2033, Fig.12a. A scan average over 37 spectra was performed and the power level at 900 Hz was set to 0 dB, Fig.12 b. In addition a ZOOM-FFT analysis of the signal was performed, Fig.12 c, whereby the power level at 900 Hz fell by 12,1 dB. The reason for this change can be found in the different bandwidths used and the different effective record lengths. The signal is a transient and therefore should be analyzed in terms of energy density, i.e. power × record length/bandwidth.

20.2 V 2KHZ 3.0 37 S120 1000.0MS

a) 10K sample Time Function T = 2s

12	



b) Scan Average Spectrum Hanning Weighting 0-2kHz $\triangle f = 5Hz$, B = 7,5 Hz N = 37; T_{eff} = 9,25K = 1,85s

c) High Resolution Spectrum Flat weighting 800 Hz - 1000 Hz $\Delta f = B = 0.5 \text{ Hz}$ T = 10 K = 2 s

14008 200HZ 5.0 37 1800 900.00HZ - 12.108R

Fig.12. Analysis of long transient signal using Scan Average with Hanning Weighting and ZOOM-FFT with flat weighting

Changing the analysis from scan average/Hanning to ZOOM/Flat causes two corrections:

- Bandwidth decreased by a factor of 15; power decreases by:
- 2) Effective record length increases by 10 K/9 ¼ K;
 power decreases by:
 0,34 dB

Total decrease of power:

The bandwidth correction above assumes that the spectrum is flat within the Hanning bandwidth.

Applications

In this section we shall show a few typical examples of the use of ZOOM-FFT in practice. We shall, however, only give a brief discussion of each case.

Gearbox Analysis

The analysis of gearboxes is an important application area of FFT analyzers. The actual working condition of the gearbox will be reflected in the vibration spectrum. Unbalance, eccentricity, and misalignment will show up as frequency components at the running speeds of the various gears and their harmonics. Uniform imperfections of the tooth profiles due to manufacture, loading or wear will give rise to components at the tooth meshing frequencies and their harmonics, and due to modulation effects, whole families of side bands can also be found around these frequencies. A detailed discussion of the use of FFT analysis in condition monitoring is given in Ref.3.

Fig.1a shows such a vibration spectrum of a gearbox, having input/output speeds of 50 Hz and 121 Hz respectively, and a tooth meshing frequency of 3150 Hz. At low frequencies the dominant peaks are found at the output shaft speed, 121 Hz, and rather unexpectedly also at its fourth harmonic, 484 Hz, actually caused by coincidence with a resonance in the shaft coupling. At high frequencies the tooth meshing is seen at 3150 Hz, while the prominent peak at 3550 Hz is most likely a so-called "ghost component", caused by imperfections in the gear cut-

ting machine.

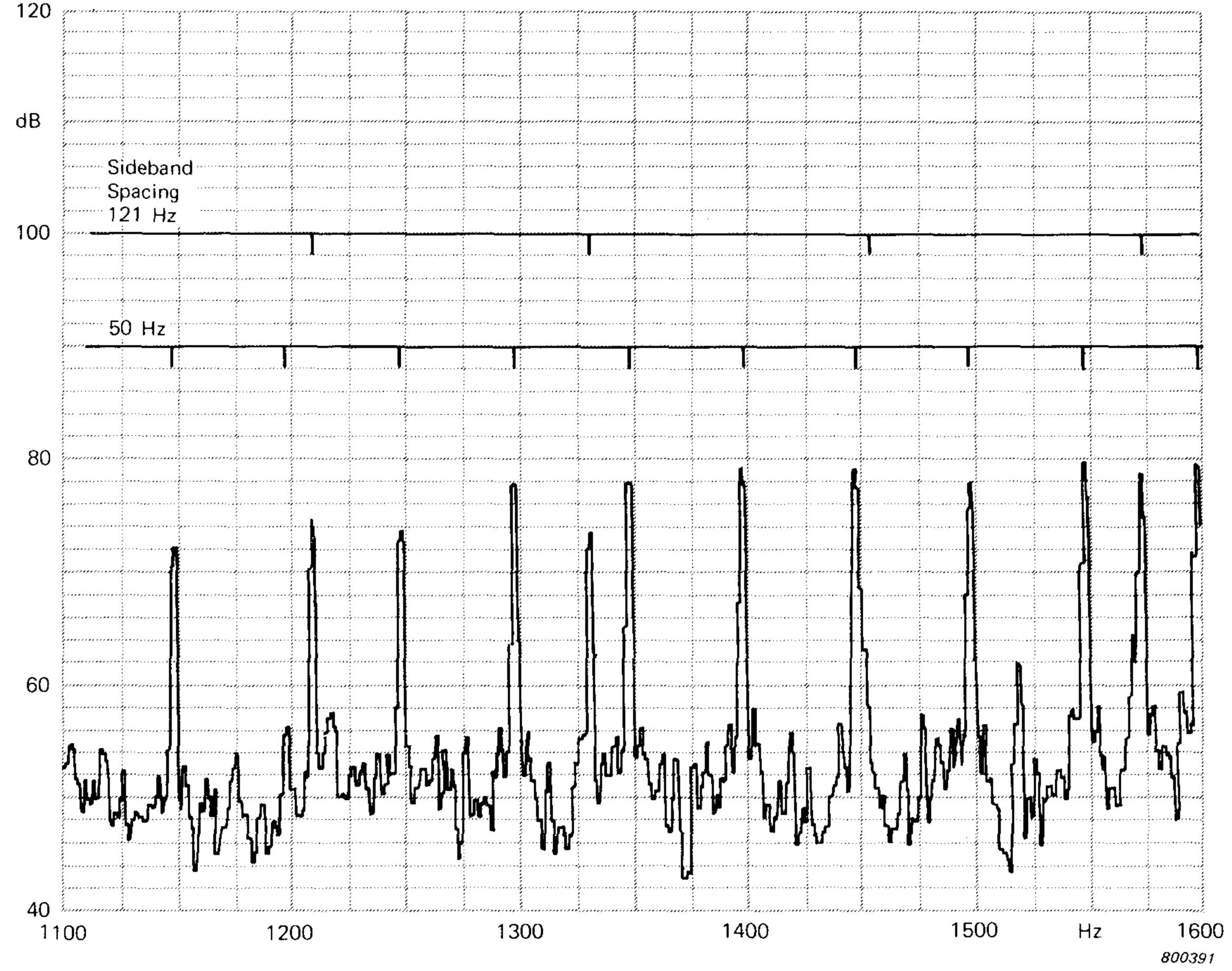
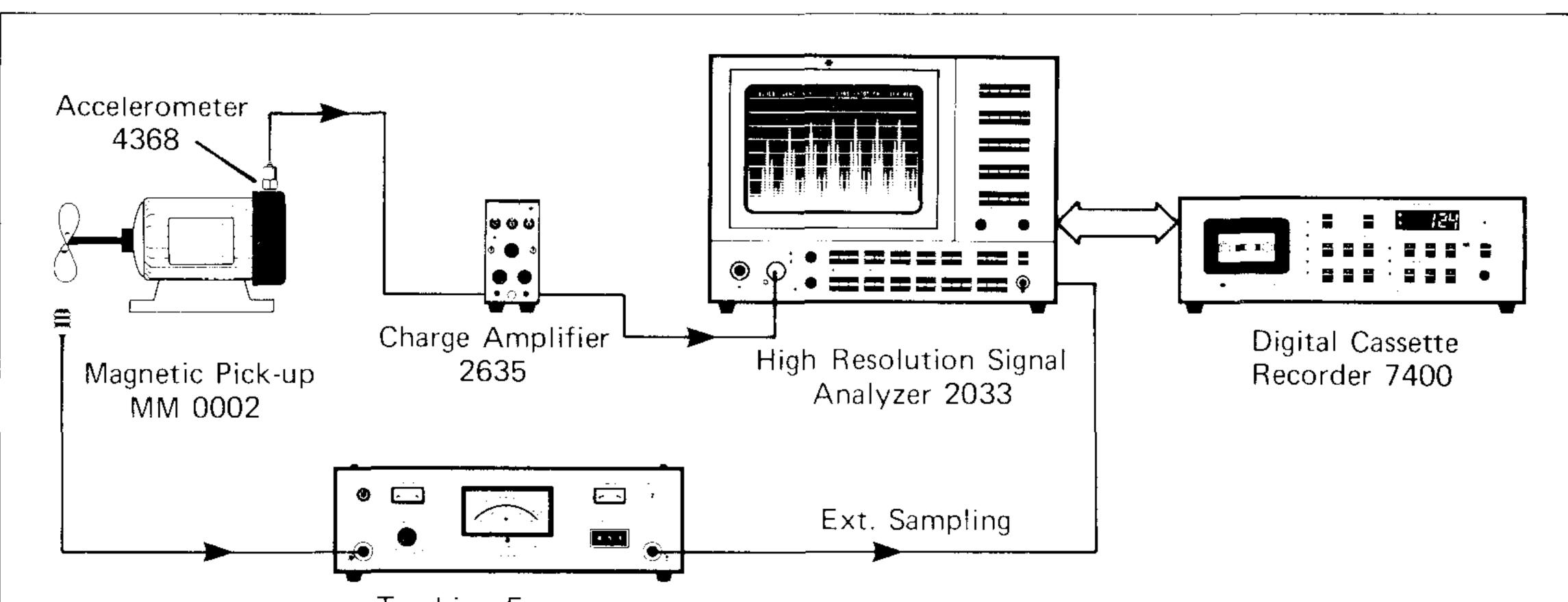


Fig.13. High resolution spectrum of gearbox vibration signal showing two families of sidebands

The finer structure of the vibration spectrum is clearly seen in Fig.1 b where a zoom factor of 10 was used to expand the region around the tooth meshing frequency. The peaks seen here belong to two families of sidebands representing modulation frequencies of 50 Hz and 121 Hz, i.e. the two shaft speeds. These sideband structures are more clearly indicated in Fig.13, where the region around 1350 Hz was expanded.

The detection of such sideband structures is most efficiently performed by use of Cepstrum techniques. Briefly, the Cepstrum is the result of an additional frequency analysis of the log power spectrum, whereby periodicities in the spectrum will be detected. This subject is discussed in detail in Ref.4. It should, however, be mentioned here that when the 2033 is used, the full 4000-line spectrum will be available for the Cepstrum analysis.

Tracking - Order Analysis The frequency range of an FFT analyzer is controlled by the sampling



Tracking Frequency Multiplier 1901

800192

Fig. 14. Use of external sampling for order analysis

frequency, i.e. $f_{max} = f_s/2,56$ as explained previously. By using an external sampling frequency, derived from the rotational speed of a machine by a frequency multiplier, the analyzer can track the machine speed and order analysis can be carried out, both in baseband and in ZOOM-FFT mode. In such an analysis the frequency range or span varies in sympathy with the machine speed such that rotationally related components stay in fixed line positions in the spectrum.

Tracking can therefore be used to prevent "smearing" effects when high zoom factors are used in the analysis of rotating machines exhibiting small speed variations. A practical set-up is shown in Fig.14. The effect of tracking is demonstrated in Fig.15, showing vibration spectra of a small electrical motor, running at a slightly varying speed due to load variations. The normal high resolution spectrum of Fig.15a exhibits considerable smearing of the components. Application of tracking external sampling, Fig.15b, reveals details not seen previously. However, the component at 250 Hz, which is related to the mains frequency is now smeared by the tracking function.

When tracking is used for order analysis of rotating machines exhibiting large speed variations, e.g. run-ups or run-downs, a new problem arises, namely the control of the analog antialiasing filter. When an FFT analyzer is used in the baseband mode, the built-in filters can only be changed in steps. During the run-up of a machine, it will be impossible to avoid aliasing completely and this will cause a reduction of analy-

sis range and of dynamic range.

However, the use of ZOOM-FFT offers another solution, where a fixed

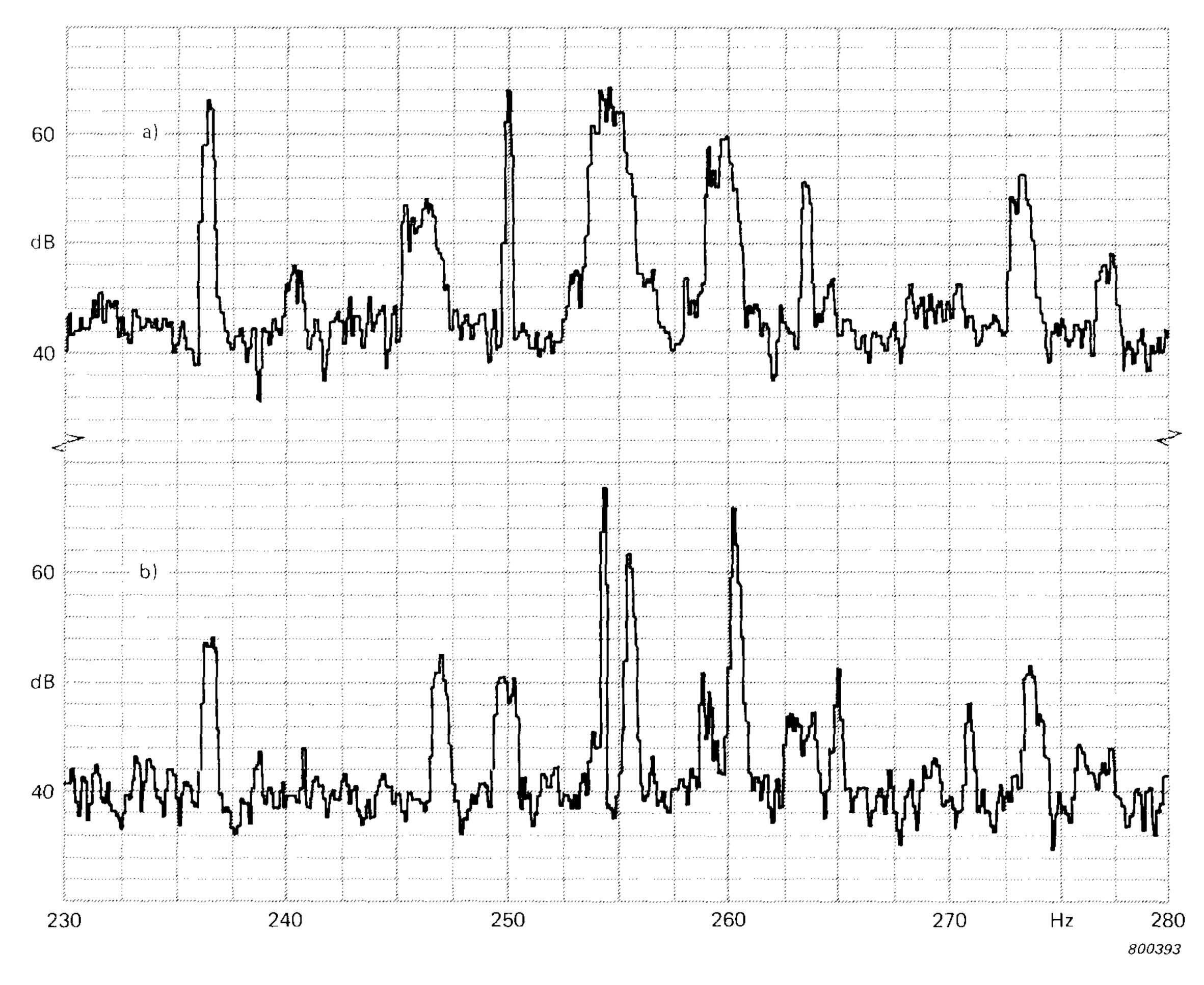


Fig.15. The effect of tracking - external sampling on high resolution spectra

antialiasing filter can be used. Let the cut-off frequency of the filter be f_f and a zoom factor of 10 be used. Hence the zoomed spectrum can be chosen to cover the frequency span from 0 to $f_f/10$, by use of a sampling frequency of 2,56 × f_f as in normal analysis. But it is now possible to increase the sampling frequency by a factor of 10 before antialiasing filter will influence the analysis, as shown in Fig.16a. But furthermore the sampling frequency can also be reduced by a factor of 1,5 before the filter characteristic folded back around $f_s/2$ starts to influence the analysis, Fig.16b.

In this case a total speed variation of a factor of 15 can be tolerated, giving a valid 400-line spectrum over the full dynamic range of the analyzer. Although the 2033 can not perform such an analysis in real-time, this is often of little consequence. A run-up of a large machine might take more than 1/2 hour, which will allow an immense amount of data to be produced. These data also have to be treated by some means during the measurement, in which case a digital cassette recor-

der, as shown in Fig.14, will provide the means of a convenient intermediate storage. A more detailed discussion of the use of FFT-techniques for order analysis will be found in Ref.5.

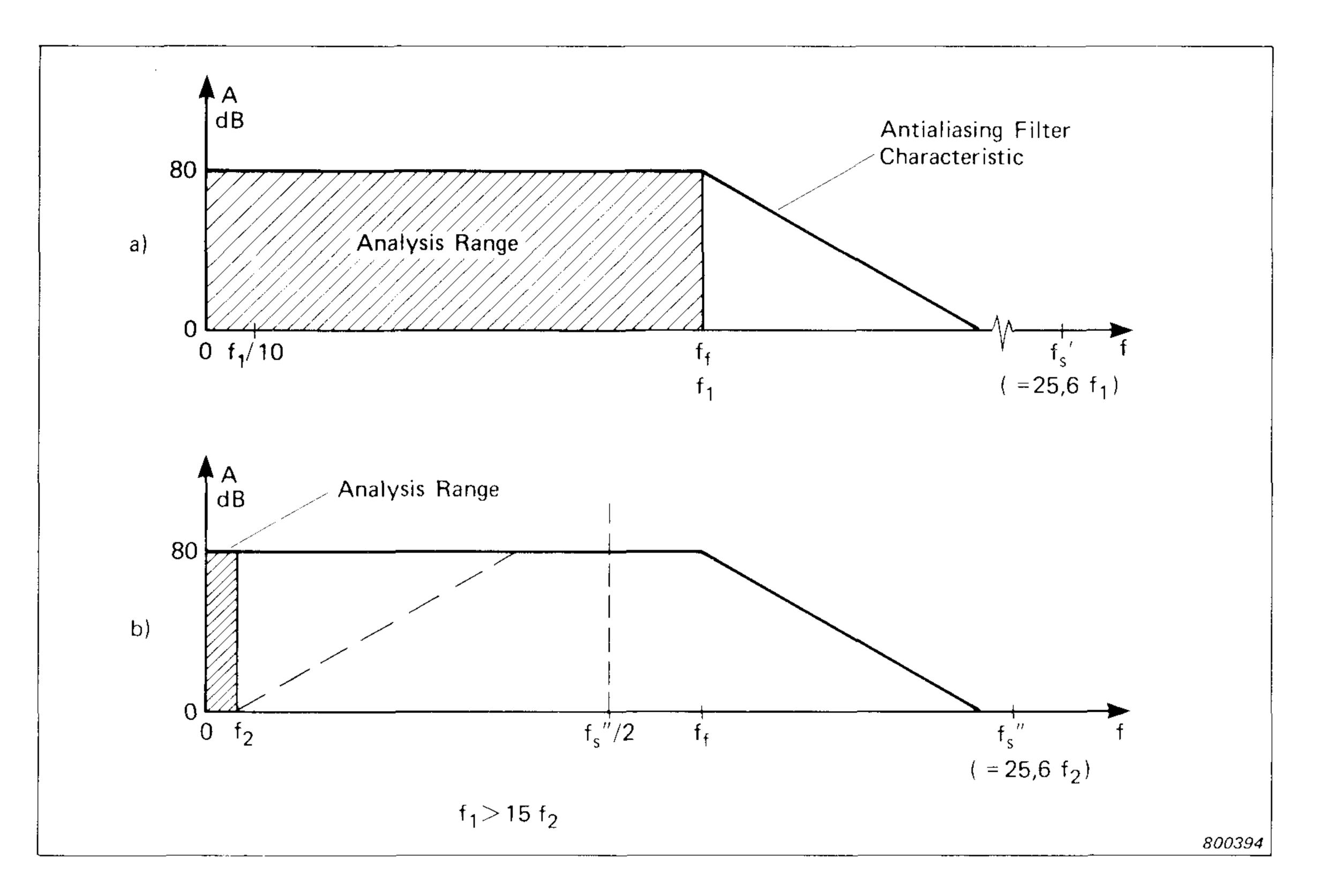


Fig. 16. Influence of fixed antialiasing filter in order analysis using ZOOM-FFT

Speech analysis

Speech analysis provides a nice demonstration of the more general analysis of non-stationary signals. The word "UTAH" (yoo'ta) was used as an example, see Fig.17. Using the 5 kHz frequency range the signal was recorded in the 10 K sample memory of the 2033, having a record length of 800 ms. The envelope of the 10 K sample signal is shown in Fig.17a. The four different sounds, i.e. (y/oo/t/a), can easily be distinguished. Using manual scan typical parts of each of the sounds were selected as the 1 K sample time record to be analyzed using the Hanning weighting. These results are shown in Fig.17b to Fig.17j.

The difference between voiced sounds, (y, \overline{oo} , \overline{a}) and the unvoiced (t) is apparent, the latter being a rather broad band noise signal. Also the voice frequency and its harmonics are prominent features of the vowel spectra. The change in voice frequency with intonation can also be seen. The difference in formants (upper envelope of the harmonic patt-

ern) of the different vowels is evident, reflecting the changes of the resonances in the vocal tract.

The use of an automatic scan on this signal produces a "slow motion" analysis of up to 2 min. duration, where the transitions from one sound to the next can be studied in detail. A scan average of this particular signal, however, would be of little practical use.

Analysis of a sonic boom

A sonic boom is the pressure wave produced by an aircraft moving at a speed greater than the speed of sound. It is often called an N-wave for obvious reasons when considering the shape of the signal as shown in Fig.18a. This transient signal has a duration of about 215 ms, and most of its energy is concentrated in the infrasound region, below 10 Hz.

The details of transient analysis can be found in the appendix. Since the signal is 215 ms long, a baseband analysis can only be performed up to 1 kHz, where the 1 K sample record length is 400 ms. Such an analysis is shown in Fig.18a and b. For high resolution analysis the signal is recorded in the 10 K sample memory, Fig.18c, having a record length of 4 s for the 1 kHz baseband frequency range. The high resolution spectrum shown in Fig.18d covers the frequency span from 0 to 100 Hz.

Two features of these analysis are worth noting (both discussed in the appendix). Firstly, observe that the maximum power level found around 3 Hz differs by about 20 dB, due to the use of different bandwidths and record lengths. Secondly, the baseband spectrum has rather poor resemblance to the true continuous spectrum due to the large ratio of signal to record length used in this analysis.

In Fig.18e is shown a recording of five contiguous high resolution spectra covering a range from 0 to 500 Hz. This spectrum is in fact the first half of the 4000-line spectrum discussed previously. The signal could also have been analyzed using scan average, in fact up to 10 kHz. However, the energy density at higher frequencies is too low to allow further details to be revealed using narrow band analysis. If the signal has to be analyzed in the audible frequency range in order to estimate the annoyance, a 1/3 octave real-time digital filter analyzer would be preferred. Here the bandwidths are much larger than those given by the FFT technique and hence the measured power levels are

10K sample Time Signal

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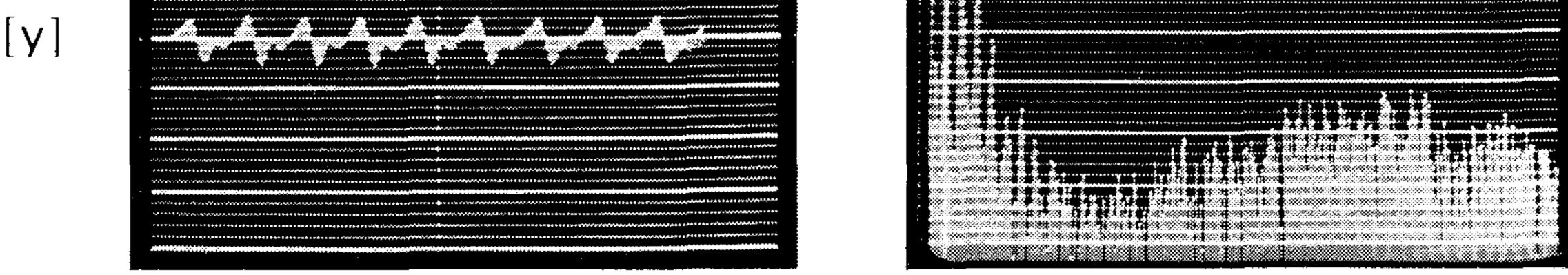
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#### Time Signal



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## Fig.17. Manual scan analysis of speech, showing different sounds of the word UTAH [yoo'tä]

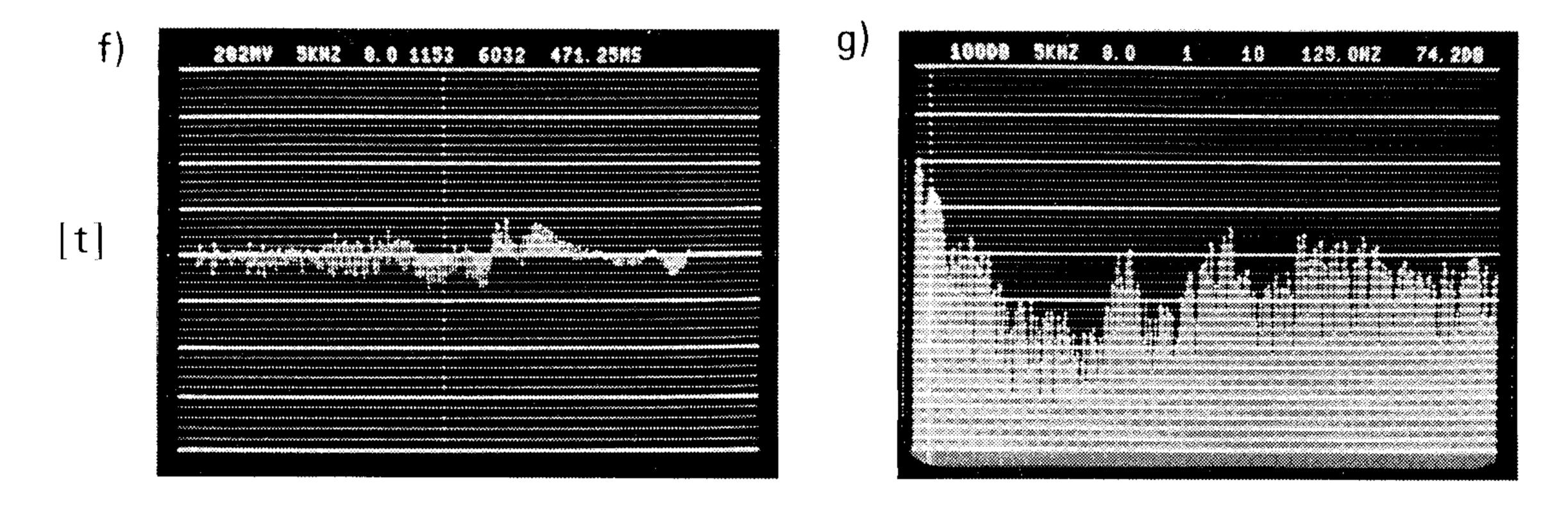
#### Time Signal

#### Spectrum

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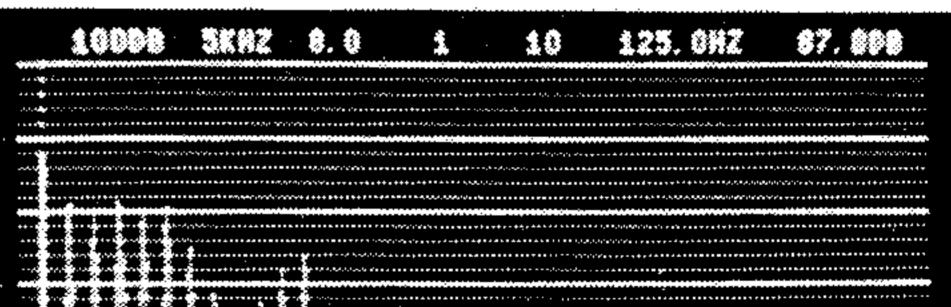
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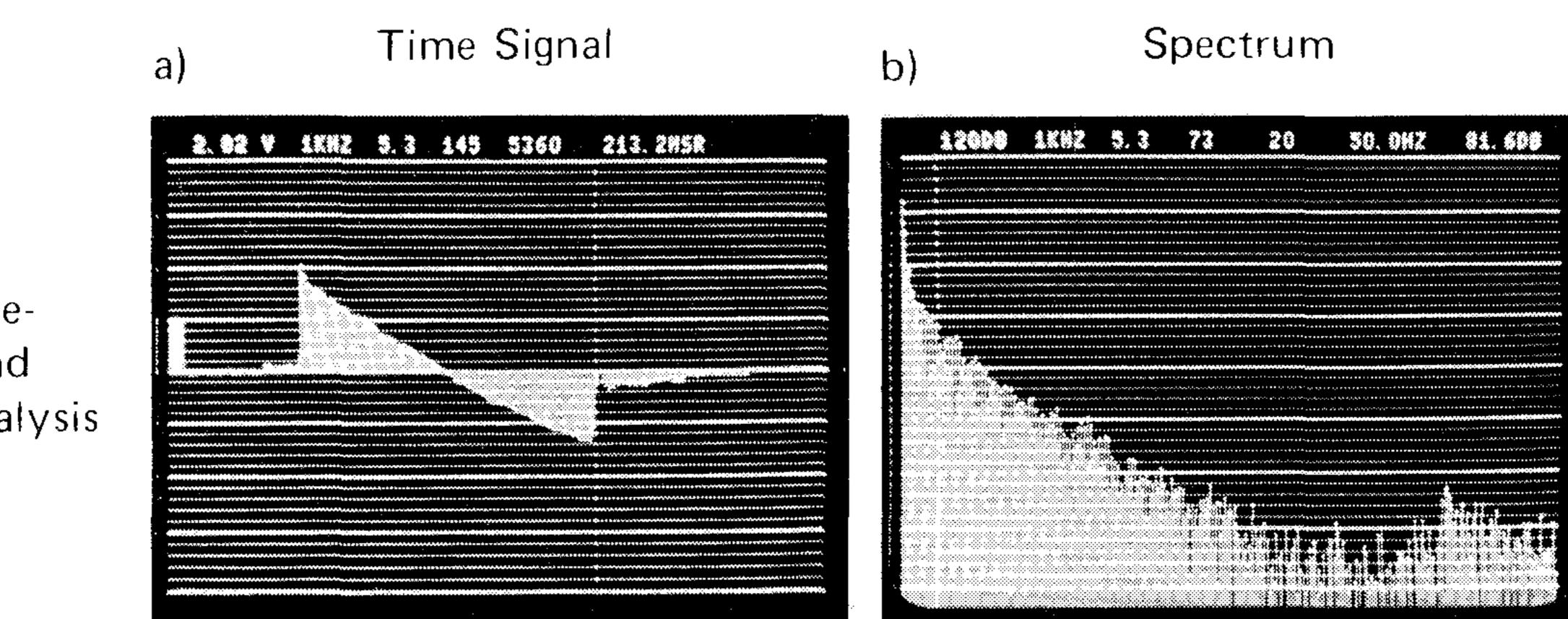
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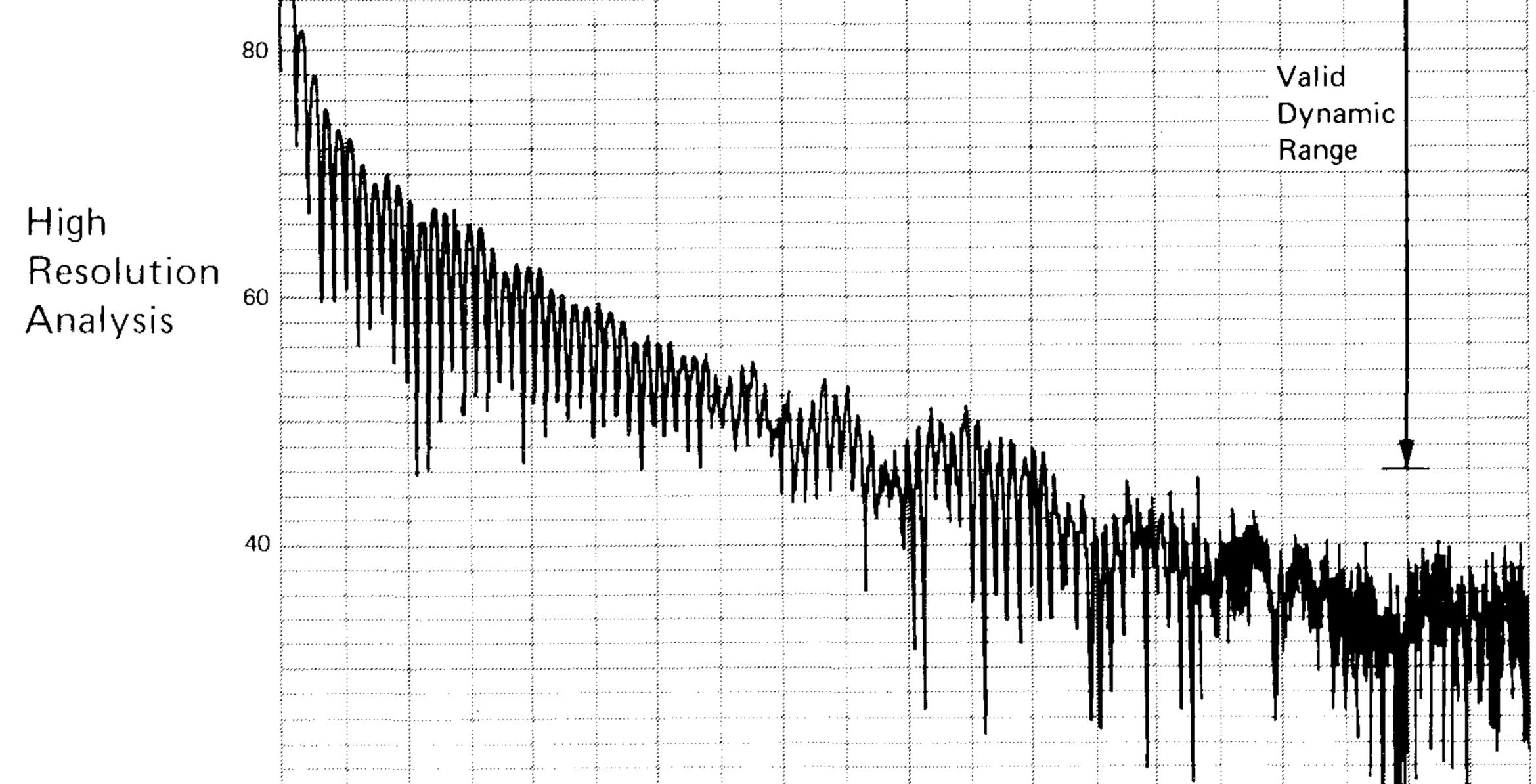
High Resolution Analysis

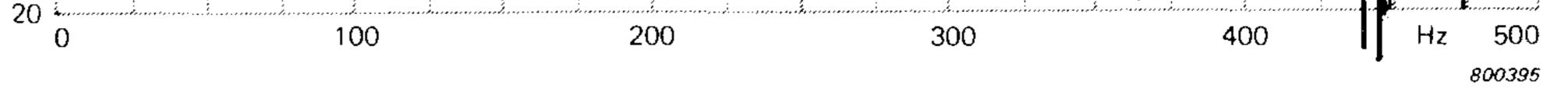
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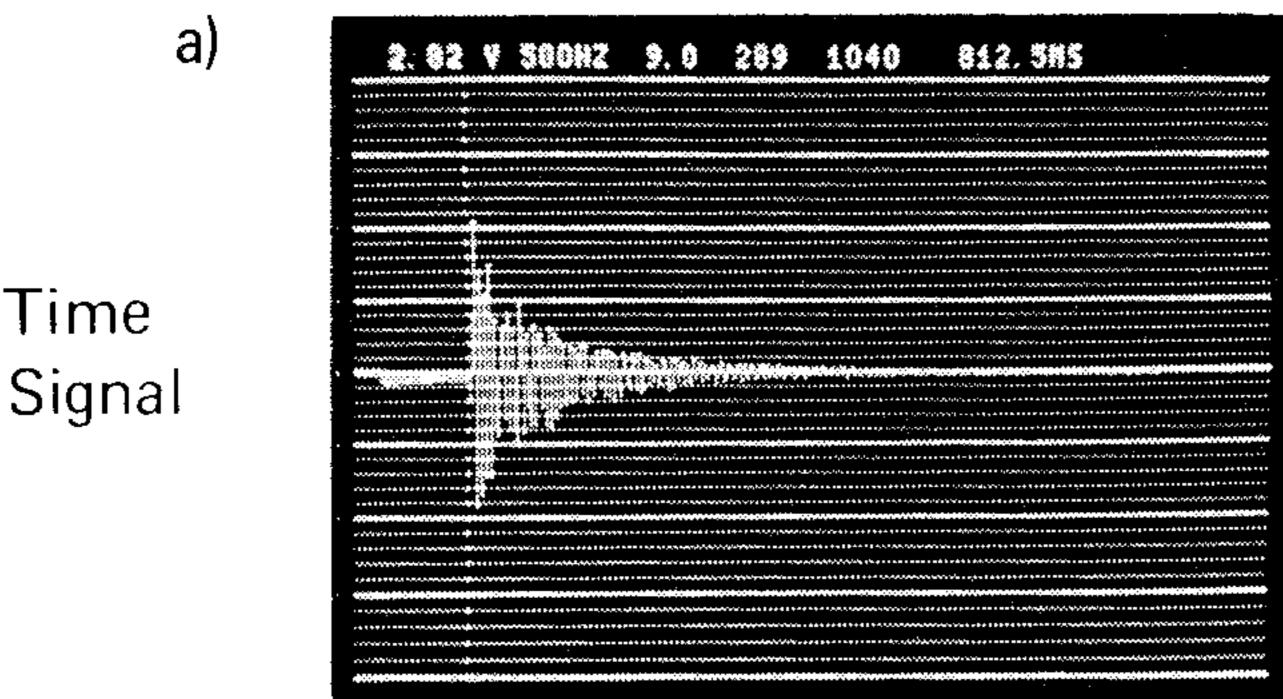
Analysis of a Sonic Boom Fig.18.

more easily brought inside the dynamic range of the analyzer. A detailed discussion of this subject can be found in Ref.6.

Modal analysis of a reverberant room

Our final example will show an application of ZOOM-FFT in acoustics. In a study of reverberation processes at low frequencies, ZOOM-FFT allowed the separation of the individual acoustical modes. This study is described in full in Ref.7.

In one part of the study the room was excited by feeding a sine-squared impulse to a loudspeaker placed in one corner of the room. A microphone was placed in another corner. The time signal recorded in the 10 K sample memory of the 2033 is shown in Fig. 19a. The 500 Hz baseband frequency range was used giving a record length of 8 s. The high resolution spectrum centered around 60 Hz is shown in Fig. 19b. Most of the spectral peaks found in the spectrum correspond to eigenmodes of the room, and could be identified by theoretical calculation, e.g. the two peaks to the left of the line selector are the axial (020) mode at 54,5 Hz and the tangential (120) mode at 58,5 Hz. The micro-



50HZ 9.9 289

Signal

6008 SOHZ 9.9 289 480 60. 000HZ - 8. 308

b)







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Analysis of acoustical modes in a reverberant room *Fig.*19.

480 60.000HZ - 7.008

C

phone was then moved to the center position of the room, and the spectrum obtained here is shown in Fig.19c. In theory all modes not having all-even indices should have a node in this point, and indeed it is seen from the measurement that the all-even (020) mode is still pres-

ent while the (120) mode has disappeared.

Conclusions

Two different types of ZOOM-FFT have been discussed. The type which is found in most FFT analyzers has the advantage of providing large zoom factors and of performing real-time analysis. However, the original time signal is not stored.

On the other hand the type of ZOOM-FFT implemented in the B&K Type 2033 does preserve the original time signal. The disadvantages of this method are that it is not real-time and that only a moderate zoom factor of 10 is available. It was argued, however, that these disadvantages were of minor importance when the practical applications of FFT analysis were considered.

In addition, the preservation of the time signal provided new analysis methods not available previously: Automatic "slow motion" analysis of non-stationary signals, scan averaging of transient signals, and fast generation of a full 4000-line spectrum, for frequency ranges from 2 kHz and down even faster than a real-time zoom analysis.



Transient analysis

In this section we shall discuss the various possibilities available in the 2033 for the analysis of transient signals, e.g. baseband, high resolution and scan average analysis. These methods differ with regard to record length and number of frequency lines being produced.

A transient signal is a signal of finite duration. An ideal Fourier transform of such a signal would give a continuous frequency spectrum, the squared amplitude indicating the energy density of the signal. Since FFT analyzers treat the recorded signal as one period of an artificial periodic signal, the squared amplitudes of FFT-spectra are power levels. However, the energy of one period, i.e. of the recorded signal, can be found by multiplying the power level, P, by the periodic time, i.e. by the record length, T. Finally, the energy density, ED, is found by dividing by the analysis bandwidth, B,

i.e.:
$$ED = P \times T/B$$
 (A1)

The physical unit of the energy density is V² s/Hz, where the V can be replaced by the amplitude unit of the physical quantity actually being measured, e.g. Pa, mm, m/s, g.

When analyzing transients using FFT-techniques, it is an obvious condition that the signal must be shorter than the record length. Furthermore, if the FFT line spectrum is to resemble the true continuous spectrum of the signal, the line spacing Δ f should be small compared to the width of any peaks in the spectrum. According to the uncertainty principle this means that the signal should be much shorter than the record length. In practice a compromise has to be made with regard to, on the one hand, apparent continuity of the spectrum and, on the other hand, the dynamic range of the analyzer and analysis time.

We shall now summarize the three analysis methods available in the 2033, the conditions to be fulfilled in order to get a correct analysis, and the parameters needed for the conversion to energy density.

Base-band analysis

400-line spectrum based on a 1 K sample time record, covering a frequency range from 0 to f_{max} .

Weighting function: Flat Record length: $T = 1/\Delta f = 400/f_{max}$

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Bandwidth:	$B = \Delta f = f_{max} / 400$

High resolution analysis

400-line spectrum based on a 10K sample time record, covering a frequency span of f_{max} /10 situated within the baseband frequency range, 0 to f_{max} . Combining ten contiguous spectra, a 4000-line spectrum can be obtained covering the entire baseband frequency range, 0 to f_{max} .

Scan average analysis

400-line averaged spectrum from a 10 K sample time record, covering the base band frequency range, 0 to f_{max} . In order not to distort the signal due to the partial analyses, the following rules should be observed:

Hanning weighting must be used and the number of spectra generated per scan must be greater than or equal to 37. The signal should be situated within the central 8K samples of the time record.

The following parameters apply to the case of 37 spectra per scan. For larger values, the effective record length is given in Table 2, however,

these variations will influence the analysis with less than 0,1 dB.

Weighting function:HanningNumber of spectra per scan:37Effective record length: $T_{eff} = 91/4 \text{ K} = 91/4 \times 400/f_{max}$ Bandwidth: $B = 1,5 \times \Delta f = 1,5 \times f_{max}/400.$

In order to illustrate these different methods, we shall analyze the rather special signal shown in Fig.A1 a under many different circumstances. The results are shown also in Fig.A1 and in Fig.A2. Note, that only the analyses shown in Fig.A1 b, A1 c, and A2 f are in agreement with the recommendations given above and will give valid results also in the case of more general signals. The other examples have been included in order to illustrate the analysis methods in more detail.

The signal shown in Fig.A1 a, is a tone burst of 3 periods of a 1 kHz sine wave, recorded in the 10 K sample input memory using a 2 kHz baseband frequency range. The signal is therefore short, 3 ms, compared to the record lengths used in the following analyses, T = 200 ms for the 1 K sample record and T = 2 s for the 10 K sample record.

Fig.A1 a only shows the central 390 samples (76 ms) of the 10 K sample record. Note that the signal starts exactly at the center of the 10 K sample record (sample 5120). This is of importance to the examples of scan average analysis given below.

This particular time signal has a specific energy density at any given frequency. Therefore, as we go through the different analysis examples, we shall give a quantitative explanation of the different power levels actually being measured. According to equation (A1), $P = ED \times B/T$.

Base Band analysis

The correct analysis is shown in Fig.A1 b, using Flat weighting. The spectrum is of the familiar sin x/x shape known for tone bursts. Although the spectrum is a discrete line spectrum, the resemblance to the continuous spectrum is very good, due to the small "signal-to-record length" ratio (3 ms to 200 ms). The peak power level, found at 980 Hz, is defined as the 0 dB level and the following analyses will be given relative to this level.

Changing the weighting function to Hanning, Fig.A1 d, results in an increase of power level of 6 dB. This is due to the position of the signal at the center of the Hanning function, where its amplitude is 2. Hence, the signal amplitude is doubled and its power therefore increased by a

Time function

b)

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Hanning

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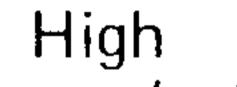
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### Baseband



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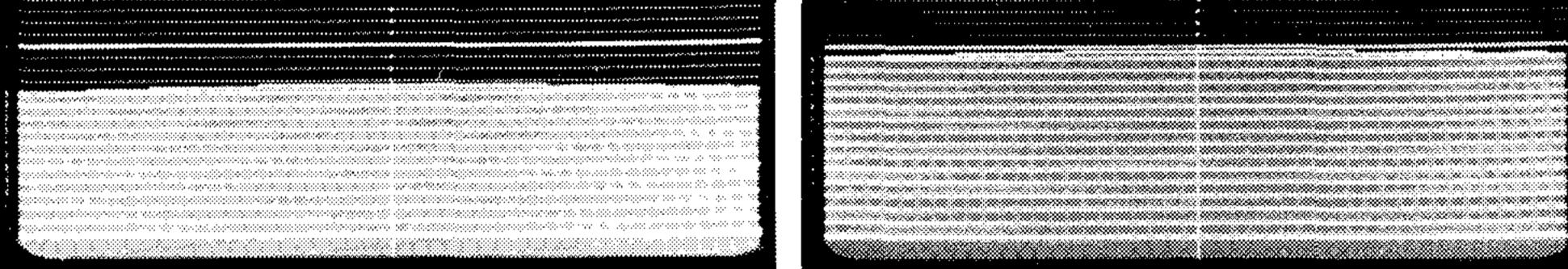


Fig.A1 Analysis of short transient signal showing differences of power levels depending on analysis mode and weighting function

factor of 4, i.e. 6 dB. Note that this correction is only valid because the signal is short relative to the length of the Hanning function.

### High resolution analysis

The correct analysis is shown in Fig.A1 c. The power level is now -20 dB. By changing from baseband to high resolution the bandwidth has decreased by a factor of 10 (-10 dB), while the record length has

increased by a factor of 10 (+ 10 dB). The total change on power level is therefore: -10 dB - (+10 dB) = -20 dB.

Changing the weighting function to Hanning, Fig.A1 e, gives a power level of —14 dB, i.e. 20 dB below the baseband - Hanning analysis of +6 dB. The explanation is in this case the same as above, since the signal is also situated in the center of the 10 K sample Hanning function.

d)

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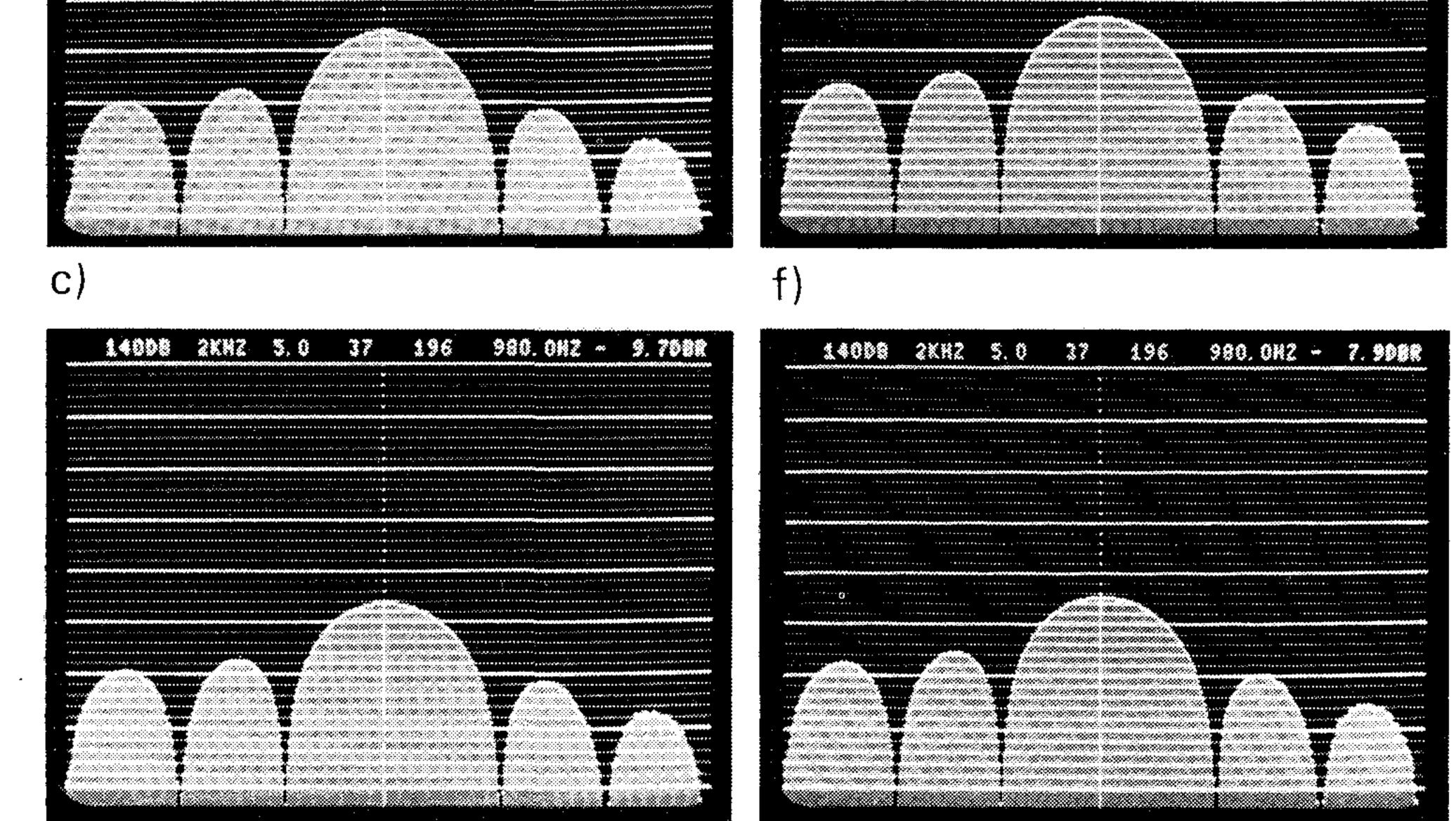
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Hanning

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### Fig.A2 Analysis of short transient signal using Scan Average. Note, Generally only the analysis shown in f) gives a valid result

### Scan average analysis

The correct analysis is shown in Fig.A2 f, where Hanning weighting is used, and the number of spectra per scan is 37. However, it will help the understanding of these conditions, to discuss also the other analyses shown in Fig.A2.

Fig.A2 a, Flat weighting, 10 spectra per scan, i.e. no overlap between the individual time records, power level:  $-10 \, dB$ . The effective length of this analysis can be shown to be 10 K samples, i.e. the record length is increased by a factor of 10, (+ 10 dB). No change in bandwidth (0 dB). Total change in power 0 dB - (+ 10 dB) = -10 dB, q.e.d.

Alternatively, the signal has been analyzed as in the baseband mode once out of 10 spectra. Hence an average value of 1/10 (--10 dB).

Fig.A2 b, Flat weighting, 19 spectra per scan 1/2 K sample overlap, power level: -9,8 dB. The effective length of this analysis is  $9^{1/2}$ K samples, an increase by a factor of 9,5 (+ 9,8 dB). No bandwidth change (0 dB) total change 0 dB — 9,8 dB = -9,8 dB. Alternatively, the signal was analyzed as in the baseband mode twice out of 19 spectra. Hence an average value of 2/19 (-9,8 dB).

This simple result is due to the very special position of the signal. Imagine that the signal had been recorded 1 ms (1 period) earlier in the memory. In this case the signal would have been cut in pieces by the Flat weighting function. Out of the 19 spectra, number 9 would have been based on only one period, number 10 on three periods, and number 11 on two periods. The resulting spectrum would have had a completely distorted shape. Hence, the smooth Hanning function should be used in a correct analysis.

Fig.A2 c probably does not need further comments at this stage. (4/37 = -9,7 dB).

Fig.A2 d, Hanning weighting, 10 spectra per scan, no overlap and complete disaster. In this case the signal is situated at the joint of two adjacent Hanning functions where the amplitude is zero (see Fig.11). Only the noise is seen.

## Alternatively, out of the 19 spectra the signal is analyzed once, with a

### Hanning amplitude of 2 (power factor of 4). Hence an average value of 4/19 (--6,8 dB).

Fig.A2 f, finally a correct analysis. Hanning weighting with 37 spectra per scan, 3/4 K sample overlap, power level: -7,9 dB. The effective length of this analysis is 91/4 K (see table 2), an increase by a factor of 9,25 (+ 9,66 dB). The bandwidth is increased by 1,5 (+ 1,76 dB).

Total change in power  $+1,76 \, dB - 9,66 \, dB = -7,9 \, dB$ . If more than 37 spectra per scan had been used, the power level would have been 

Although we actually could account for all the different power levels measured in these examples, we should remember that the signal used was of a rather academic type. When dealing with practical signals only the three correct types of analysis will allow us to do the conversion to energy density units.

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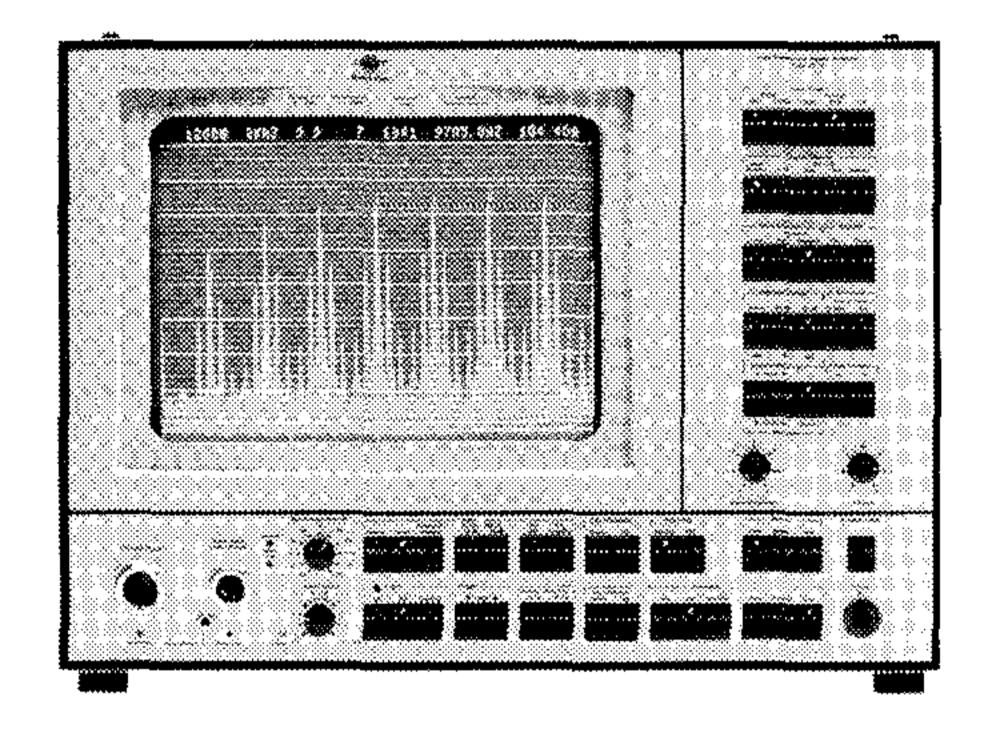
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### News from the Factory

### High Resolution Signal Analyzer Type 2033



The High Resolution Signal Analyzer Type 2033 consists of a combined transient recorder and a Fourier Analyzer. The 10K sample memory of the transient recorder with an extremely flexible trigger greatly enhances the capability of the 2033 in the analysis of transient and non-stationary signals. Further, a unique zoom feature which preserves the time function allows a 4000 line spectrum to be generated from a single time record, in contrast to the usually implemented zoom transforms which require the time function to be re-recorded for each new zoom frequency range.

The analyzer has two modes of operation, *baseband* and *high resolution*. In baseband mode the 2033 operates like a conventional 400-line FFT analyzer by sampling the input signal and transforming it, 1 K samples at a time, into the frequency domain. The spectrum produced by each transformation (baseband spectrum) is a constant bandwidth spectrum measured at 400 equally spaced frequency intervals, across a frequency range (baseband) which is selectable in a 1 - 2 - 5 sequence from 0 Hz (nominal) to 10 Hz through 0 Hz (nominal) to 20 kHz. The sampling frequency is always 2,56 times the selected baseband frequency range.



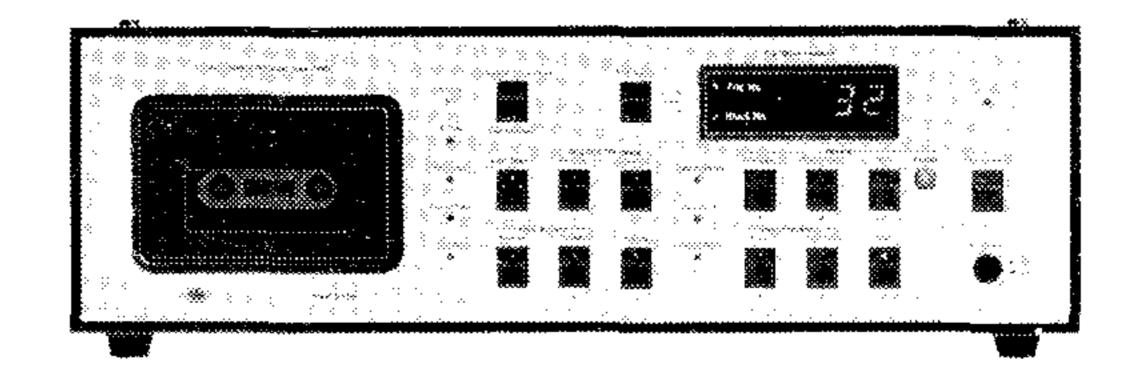
In high resolution mode the 2033 operates on 10K samples (instead of 1 K) which can then be used to produce a 10 times "zoom" whereby part of the baseband frequency range is expanded by a factor of 10 such that the entire 400 lines of the spectrum lie within a frequency range which is one tenth of the baseband frequency range. This zoom is applied around the line selector which can be positioned anywhere within the baseband frequency range.

Another method of analysis is a *scan* analysis in which 10 K samples of the input signal are recorded and stored in the input memory. A time window 1 K samples long can then be automatically or manually stepped along the 10 K sample time function and analyzed after each step. This method of analysis gives the possibility of "scanning through" the 10 K sample time function to give a "slow motion" analysis, allowing any changes in the baseband spectrum to be observed as the time window is stepped along.

Both the baseband and high resolution spectra can be linearly or exponentially averaged or their maxima can be stored. These spectra as well as the instantaneous spectrum and the time function can then be displayed on an 11" calibrated display screen, and the displayed values read via a line selector.

Output of displayed data can be made to an X-Y or Level Recorder. The 2033 is also equipped with an extremely flexible and sophisticated IEC 625-1/IEEE Std. 488 compatible interface which allows simple connection to a desk-top calculator. The calculator then has access to all the pushkeys, time function, complex spectrum, instantaneous 2033 power spectrum, averaged power spectrum, stored spectrum, and the display screen alphanumeric text line. Further processing of the data can then be carried out to obtain automatic spectrum comparison, Inverse Fourier Transform, Cepstrum analysis, and Hilbert Transform.

Digital Cassette Recorder Type 7400



The proliferation of measuring and analysis equipment using digital techniques has created a greater need for long-term storage of data in digital form. With the new B & K Digital Cassette Recorder Type 7400, data can be stored on magnetic tape cassettes which are very attractive owing to their small size and sheer convenience in use.

The 7400 can record and playback a total of approximately 500 kbytes of data on a Philips Compact type digital cassette at a speed of 15 ips. Recording is in accordance with the ECMA 34/41 and corresponding ISO, BS, DIN and ANSI standards. The number of spectra which can be stored on one cassette is in the order of 100 narrowband (400 line)

spectra from the Spectrum Analyzer Type 2031 or several hundred octave or third octave spectra from the Digital Frequency Analyzer Type 2131.

Extensive search facilities are built-in to allow files and blocks of data to be rapidly located on the tape. During recording and playback a checking function is in continual operation to insure the integrity of the data transfer to and from the tape. The average maximum data transfer rate is 1 kbytes/s.

Data transfer to and from the 7400 is via the IEC 625-1/IEEE 488 interface bus or B & K low-power interface bus which are built-in. This permits communication with a wide range of equipment using these interfaces including the following B & K equipment:

Spectrum Analyzers Type 2031, 2033 and 2131. Sound Power Calculator Type 7507. Alphanumeric Printer Type 2312. Strain Measuring System Types 1526, 1544 and 1545. Noise Level Analyzer Type 4426, and Impulse Precision Sound Level Meter Type 2210.

Power can be provided from an AC mains supply or from an external 10 to 30 V DC supply, for example a car battery.

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(Continued from cover page 2)

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Frequency Analysis (English)

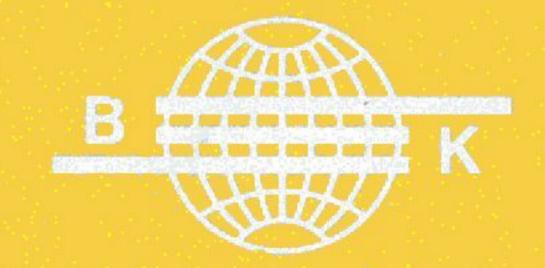
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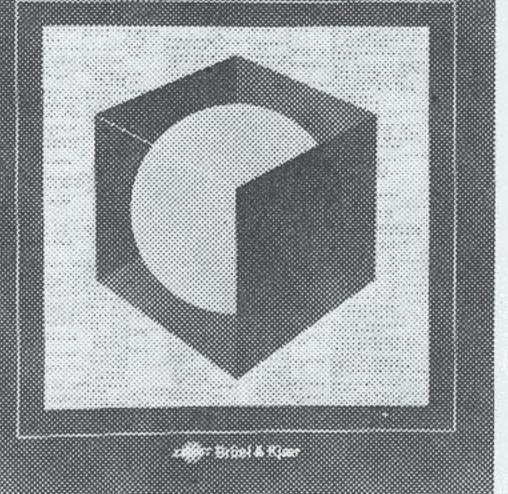
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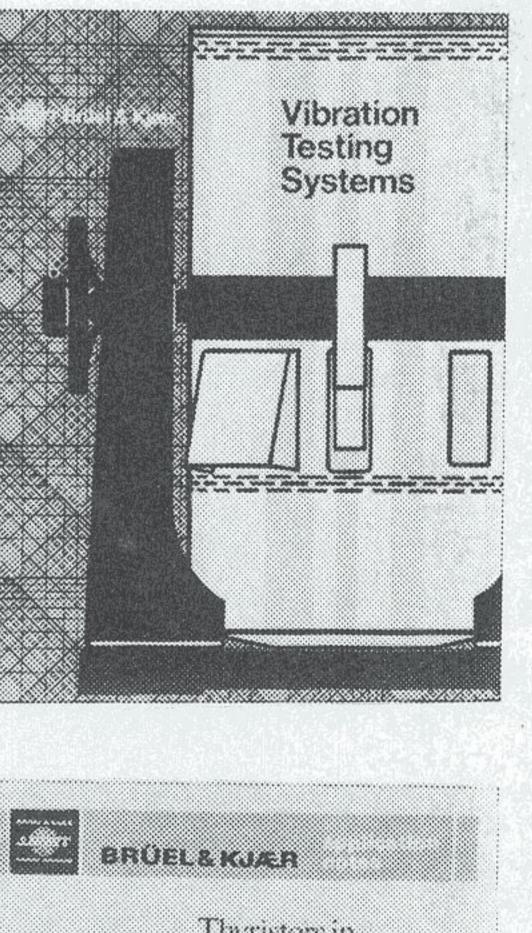


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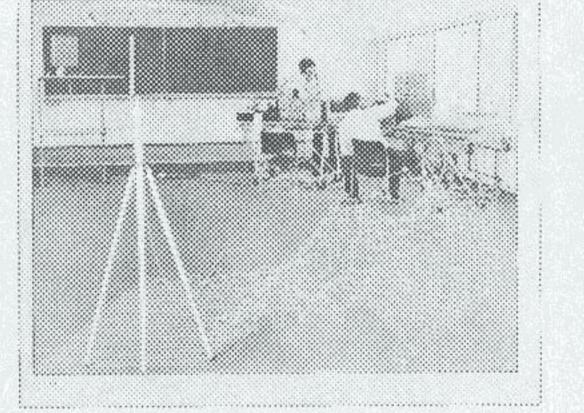
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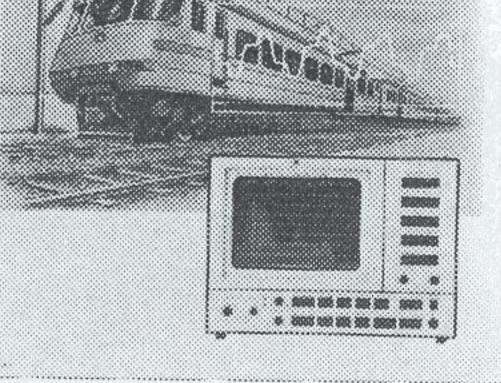
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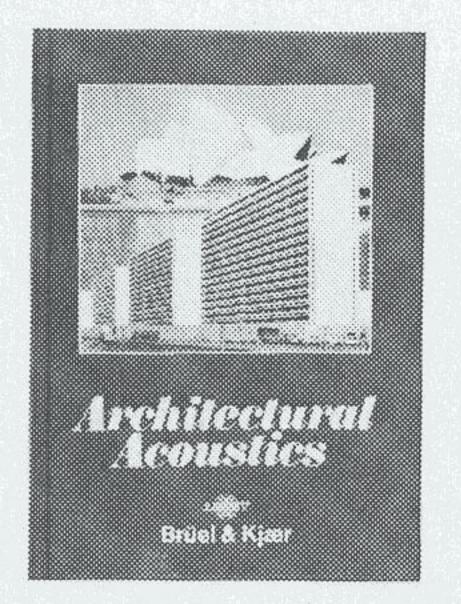




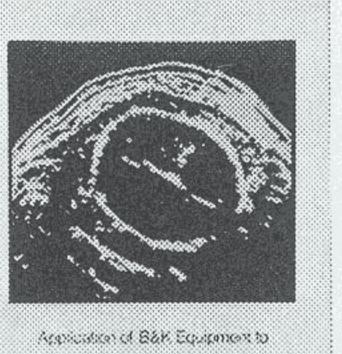
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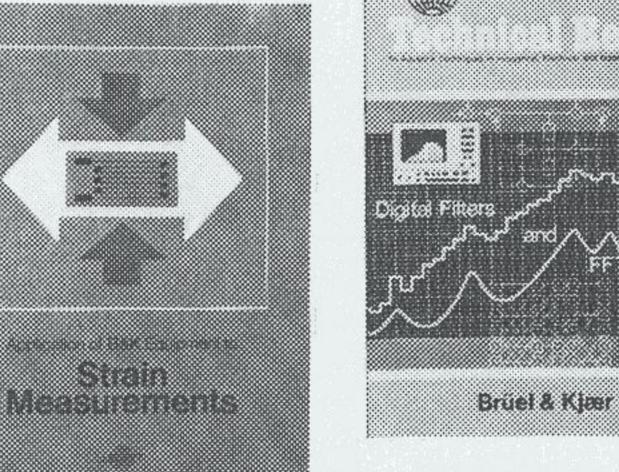




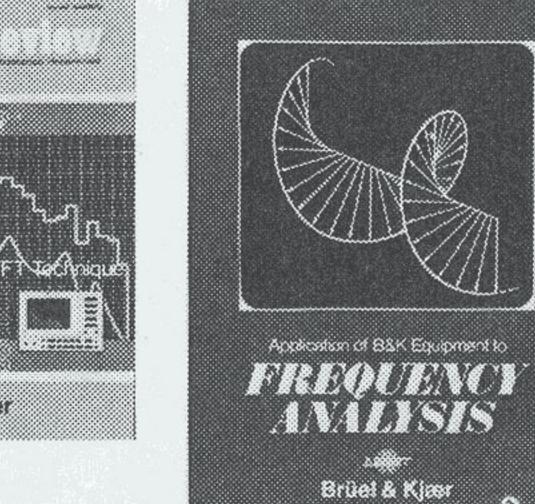
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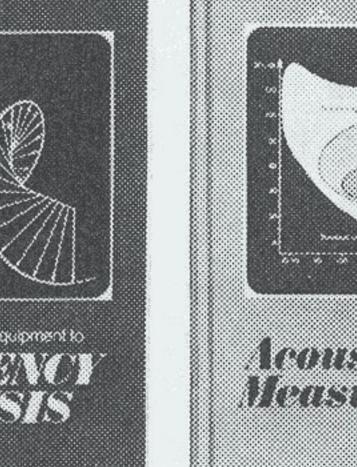


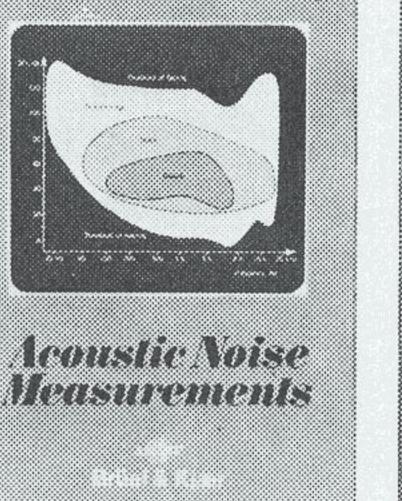
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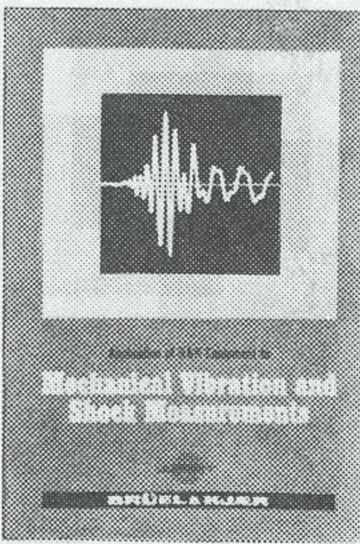


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